Physics 471 – Introduction to Modern Atomic Physics
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**HOMEWORK I** (Due: 10/20/2006)

**(Please answer 6 of the following 10 questions.)**

**1. Quantum states in a 3D Gaussian potential**

Consider $H = \frac{p^2}{2m} - A\exp(-2r^2 / r_0^2)$, where $A$ is a constant.

A. How large should *A* be to support a bound state?
B. When *A* is much larger than that value, estimate how many bound states are there?
C. Following B., what are the densities of states at the center and near the rim?

**2. Creation of coherent state and squeezed state**

An oscillator, described by $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2$ is initially in the ground state.

A. At $t = 0$, we instantaneously displace the trap by *a*, show that the particle is transferred to a coherent state $|b>$ in the new trap.
(Hint: show $\hat{a}|b> = b|b>$ and determine b.)
B. Determine $|b(t)>$.

**3. Phase-shift due to Landau-Zener probe (Difficult)**

Consider $H = \begin{bmatrix} Pt & \hbar\Omega \\ \hbar\Omega & 0 \end{bmatrix}$, where $P = const.$ and *t* is time. At $t = -\infty$, the system is in the zero energy state $|t = -\infty> = |E = 0>$. Determine $|t = +\infty>$ to leading order in $\Omega << \sqrt{P/\hbar}$.

**4. Ramsey spectroscopy**

Derive the Ramsey lineshape showed on page 19 of Lecture 1 based on the scheme shown on page 14 of Lecture 2.

**5. Dark state**

Construct a dressed state basis $|A; N_1, N_2>$ for the three-level system on page 17 of Lecture 2, where A=a, b or c is the quantum number of the atoms, $N_1$ and $N_2$ are the photon numbers of the two radiation modes.

A. What are the dark states in the following cases: 1. $\Omega_a \neq 0$ and $\Omega_b = 0$, 2. $\Omega_a = 0$ and $\Omega_b \neq 0$ and 3. $\Omega_a \neq 0$ and $\Omega_b = 0$.
B. In the third case, what are the other two “bright” eign-states?

**6. Atomic structure**

a. Why is periodic table filled in the order of 1s 2s 2p 3s 3p 4s 3d 4p 5s…?
b. Use mathematica to plot out one of the angular wave functions shown on page 4.

c. Consider an *nsnp* configuration, sketch the energy scheme from LS coupling to jj coupling. (Hint: Compare page 6 and page 13 of Lecture 3).

**7. Zeeman shifts in the $^6$Li ground state**

Derive the eigen-energies in the ground state hyperfine manifold of 6Li in the presence of a magnetic field B. (Hint: See Page 11 of Lecture 3 or you can check http://www.phy.duke.edu/research/photon/qoptics/techdocs/pdf/PropertiesOfLi.pdf.)

**8. Fine structure of atoms**

Consider the 2p state of a hydrogen atom in the absence of external fields. Spin-orbit interaction is given by $H_{S-O} = A\, s \cdot L$

A. Express the eigen-energies and eigen-states in terms of $| s = 1/2, L = 1; m_s, m_L >$

B. If you wish to radiatively couple $| 1s_{1/2}, m_j = 1/2 >$ to $| 2p_{3/2}; m_j = 3/2 >$, how would you orient the electric field (polarization) of the laser beam (Hint: Choose Z axis as your quantization axis.)

C. What is the ratio of the following dipole transition strengths:
$| 1s_{1/2}, m_j = 1/2 > \to | 2p_{3/2}; m_j = 1/2 >$ and $| 1s_{1/2}, m_j = 1/2 > \to | 2p_{1/2}; m_j = 1/2 >$?

D. Now consider a carbon atom in the configuration of $2p^2$. Determine the terms and eigen-states of the fine structure in the LS-coupling scheme.

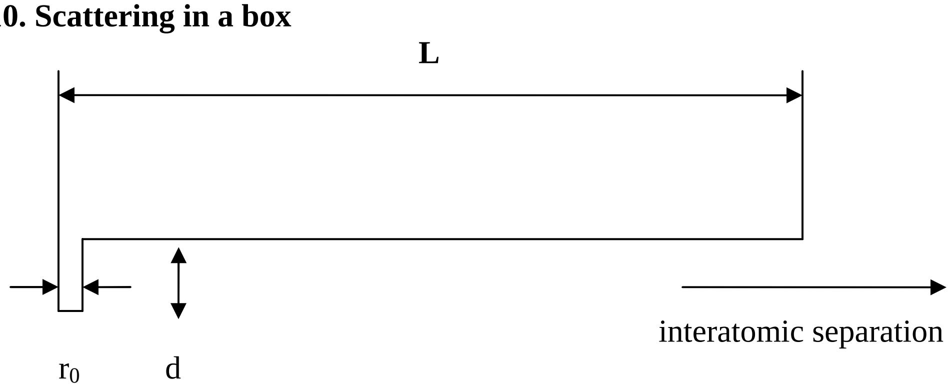
**9. Thermal gas in a harmonic potential**

Consider a harmonically trapped gas, $H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 r^2$, at temperature T.

A. What are the distributions of density and momentum of the gas.

B. Repeat A in a linear potential $H = \frac{p^2}{2m} + Ar$.

**10. Scattering in a box**




Approximate the continuum by a large box with size L and the interaction range is $r_0 << L$.

A. What is the critical $d = d_c$ when the 1st bound state forms in the interaction potential?

B. Sketch the ground state wave function of the system at $d = 0.99 d_c$.

C. Sketch the bound state and the ground state in the "continuum box" at $d = 1.01 d_c$.

D. Take $L / r_0 \to \infty$, determine scattering length $a$ at $d = 0.99 d_c$, $1.00\, d_c$ and $1.01\, d_c$.