

**HOMEWORK II**

(Due: 11/08/2006)

**(Please answer 6 of the following 10 questions.)**

**1. Stability of ions in a Penning trap**

Consider a Penning trap with an electric potential of  $\phi(x, y, z) = \frac{A}{2R^2}(2z^2 - x^2 - y^2)$

and a magnetic field of  $B = B\hat{z}$ . An ion with charge  $q > 0$  initially rests at  $(x_0, y_0 = 0, z_0)$ .

- Show that the equation of motion satisfies  $\ddot{\vec{r}} = (\bar{\omega}_c \times \dot{\vec{r}}) + \omega_s^2 \vec{r}$ . Determine  $\bar{\omega}_c, \omega_s$ .
- What are the stability conditions?
- Can you model the magnetron motion with an effective potential. (See slide 9 of lecture 5 and slide 4 of lecture 6).

**2. Linear Paul trap**

Consider a linear Paul trap with  $\phi(x, y, z) = A \frac{(x^2 - y^2)}{2R^2} \cos \Omega t + \frac{A}{2Z^2}(2z^2 - x^2 - y^2)$ ,

where  $R(Z \gg R)$  is the characteristic length scale in the radial(axial) direction and A is a constant.

- Show that the radial motion satisfies the Mathieu equation  $\ddot{x} = -(\omega^2 - 2q_x \cos \Omega t)x$ .
- Show that for small  $q$ , the solution is approximately  $x = x_0 \cos(\omega_c t + \phi)(1 + q \cos \Omega t)$ .
- Are the  $\omega_c$  and  $q$  from B. compatible with the physics picture on slide 4 of lecture 6?

**3. Stability of the Mathieu equation (See slide 5 of lecture 6)**

There are multiple stable regimes in the parameter space. Provide a picture to understand one or more of the following observations

- Stable regimes form long skinny fingers for large  $qs$ .
- for  $a > 1$ , only “quantized” values of  $q$  lead to stable dynamics.
- for  $q \ll 1$ , only “quantized” values of  $a$  lead to unstable dynamics.

**4. Achieving  $\Delta m / m = 10^{-8}$  based on inducing a dynamical instability?**

Use the model in 2., and the numerical calculation in slide 15 of lecture 5 as a guide.

Consider two ions with the same electric charge and masses of  $m$  and  $(1 + 10^{-8})m$ .

- how well do you have to control  $A, R, \Omega$  so you can distinguish the two ions?
- assume  $q$  can be precisely tuned to destabilize one ion, what is the time scale (in the unit of  $1/\Omega$ ) to distinguish the two ions?

**5. An ion or atom trap with only static E field, no B-field or laser field?**

Earnshaw’s theorem: no 3D maximum can be created in free space based on static fields.

- Find a configuration which can create an E-field maximum in 2D.
- In the third (z-)direction, can we use, say, gravity +E-field to create a local minimum?
- If it is possible, is it realistic?  
(Hint: Assume E field can be as strong as  $10^7$ V/m, what would be the potential depth?)

### 6. Photon recoil of a classical particle in free space

A free atom with mass  $M$  and momentum  $\vec{P}$  in the ground state absorbs a photon with momentum  $\vec{p}$  and then decays back to the ground state by emitting a photon with  $\vec{p}'$ .

A. How much is the energy transfer to the atom,  $\Delta E(\vec{P}, \vec{p}, \vec{p}')$ ?

(Hint:  $E = P^2/2M$  and  $\Delta E = (p - p')c$ .)

B. Assuming  $\vec{p}'$  is unidirectional, what is  $\langle \Delta E(\vec{P}, \vec{p}) \rangle$  and the meaning of each term?

C. Assuming  $\vec{p}$  is also unidirectional, what is  $\langle \Delta E(\vec{P}) \rangle$  and its physical meaning?

### 7. Photon recoil of a two-level atoms in a harmonic trap

A 1D harmonically trapped 2-level atom interacting with a monochromatic radiation can be modeled by the Hamiltonian shown on slide 10 of Lec. 6.

A. What is the absorption spectrum in the lowest vibrational state  $|g; \nu = 0\rangle$ ?

(Hint: Evaluate  $\langle k | H_I(\Delta) | 0 \text{ or } 1 \rangle$  for  $k = 0, 1, 2, \dots$ , you can assume  $\eta \ll 1$ .

$\eta \ll 1$  is the Lamb-Dicke regime, meaning the oscillator length of the harmonic trap is small compared to the wavelength of the radiation.)

B. What is the emission spectrum of an excited atom in the ground state  $|e; \nu = 0\rangle$ ?

C. Show that the average energy transfer in A. and B. is the same as in problem 6.

### 8. Levitation of a glass ball

A transparent glass ball with index of refraction  $n$ , radius  $r$  is illuminated by a laser with a uniform intensity  $I$ . Show that the radiation force is  $F = \alpha I r^2$  and determine  $\alpha$  to 20%.

### 9. Doppler cooling limit (see slide 6 of lecture 8)

A. Find the condition for the maximum damping rate (consider only low intensity limit).

B. Under this condition, find the heating rate due to spontaneous emission?

C. What is the equilibrium temperature?

(Hint: Equilibrium temperature is reached when cooling=heating.)

### 10. Random walk model of Raman cooling

In a very simple picture, Raman cooling can be modeled as a random walk process in the momentum space. Assume atoms are randomly kicked in a 3D momentum space with a step size of  $\Delta p = \hbar k$  until they reach the center (dark state):  $|p| < \Delta p$ .

A. Given a free gas with an initial thermal energy of  $kT = N(\Delta p^2 / 2m)$ ,  $N \gg 1$ , what is the fraction of the atoms that can eventually reach the center?

B. What is the time scale to capture those atoms?

C. At that time scale, what is the temperature of the uncooled atoms?

(Hint: model the atoms outside the center as a thermal sample.)