Physics 471 – Introduction to Modern Atomic Physics
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HOMEWORK IV (Due: 12/4/2006)

(Please answer 5 of the following 10 questions.)

1. Production of molecules in the thermal sample of weakly interacting atoms

Molecules can be created by the reaction $A+A\leftrightarrow A_2+\Delta E$, where ΔE is the released energy. Verify the result on slide 14 of Lecture 13, namely $N_M / N_A = \phi_A e^{\Delta E/kT}$.

(Suggestion: Minimize the free energy $F = -kT \ln Z_{sys}$, where the partition with N_A>>1 atoms and N_M>>1 molecules is $Z_{sys} = \frac{Z_M^{N_M}}{N_M!}\frac{Z_N^{N_A}}{N_A!}$, $Z_M = Z_N e^{-\Delta E/kT}$ and $N_A + 2N_M = const.$)

2. Molecules or atoms in a dipole trap?

Energy is conserved in a dipole trap.

A. Prepare a pure atom sample at $T=T_0$, determine N_M / N_A in equilibrium.

B. In the limit when $\Delta E/kT_0 \rightarrow \infty$, do you expect atoms of molecules in the trap?

3. Mini BEC-BCS crossover in a harmonic trap

Show that the Hamiltonian of one spin-up and one spin-down fermionic atoms is $H = H_1 + H_2 + V(|r_1 - r_2|)$, where $H_{i=1,2} = p_i^2/2m + \frac{1}{2}m\omega^2 r_i^2$ and V is the interaction.

A. Show that $H = H_{CM} + H_r$, where $H_{CM}(H_r)$ is the center of mass (relative) Hamiltonian.

B. What is the total energy of the system at zero temperature in the non-interacting regime (BCS limit)?

C. What is the total energy in the molecule regime (scattering length a is positive and small compared to the oscillator length $0 < a << \sqrt{\hbar / m\omega}$)

(Hint: Assume V is short range, $a = -\frac{\psi(r)}{\psi'(r)}$ evaluated at $r=|r_1-r_2|=0^+$.)

D. What is the system energy when $a=\pm\infty$ as compared to that with no interaction?

4. Dissociating molecules into entangled atom pairs

One way to "see" the molecules is by dissociating molecules back to atoms. Assuming a diatomic molecule is at rest with the molecular wavefunction $\psi_m(r = |r_1 - r_2|)$. At t=0, we suddenly bring its energy far above the continuum to E>0. The molecule then dissociates into two atoms and what is the wavefunction of the atoms $\psi(r_1, r_2, t)$?

5. N-qubit Greenberger-Horne-Zeilinger state

A GHZ state in a N-qubit system is given by $|GHZ\rangle = 2^{-1/2}(|\downarrow\downarrow\downarrow\downarrow\ldots\rangle + |\uparrow\uparrow\uparrow\uparrow\ldots\rangle)$

A. Write a quantum circuit to construct a two-qubit GHZ state starting from $|\downarrow\downarrow\rangle$.

B. Write a quantum circuit to construct a three-quibit GHZ state starting from $|\downarrow\downarrow\downarrow\rangle$.

C. What is the unitary transformation U which converts $|\downarrow\downarrow\downarrow\rangle$ to the GHZ state?

D. Estimate how many quantum logic gates you need to construct an N-qubit GHZ state.

6. Quantum Apples

Apples are produced in two states, |edible> or $2^{-1/2}$ (|edible>+|poisonous>). Assume you can perform any quantum operations (shown in lecture 15) on the apples.

A. If you randomly pick up an apple and eat it, what is the chance you are not poisoned?

B. If I lend you some |edible> apples, can you quantum compute whether yours is edible?

C. If I lend you a |poisonous> apple, can you make yours edible?

D. Now you want to poison Snow White, you take two $2^{-1/2}$ (|edible>+|poisonous>) apples. How do you configure the apples s.t. she will definitely die after eating them?

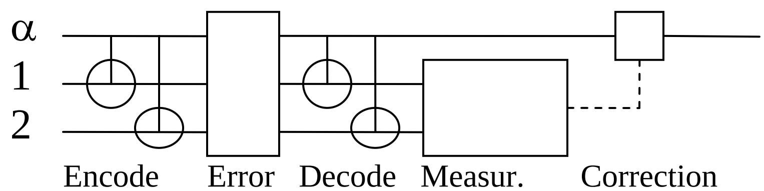
7. Quantum teleportation

Adopt the protocol on slide 21 of lecture 15 and the Bell state basis on slide 15,

A. Express $|\phi >=|S>_{13}\otimes|\alpha>_2=\sum_k|\psi_k>_{12}U_K|\alpha>_3$ and determine U_K.

B. If the measurement shows that 1 and 2 are in the GHZ state. How do you reconstruct the state |α>?

8. Simple quantum error correction



Assume a qubit is in the state of |α> = a |0>+b|1> and the ancllae are initially |0>.

A. Show that the encoding stage creates | >=a|000>+b|111>.

B. Assume an error is introduced as an addition of ε| >' to the state. What do you get from the decoding stage when | >'=|100>, |010> or |001>?

C. Say, | >'=|100>, list all possible results from Measur. and the state of the qubit?

D. How do you bring back the quantum state of the qubit? How small should ε be?

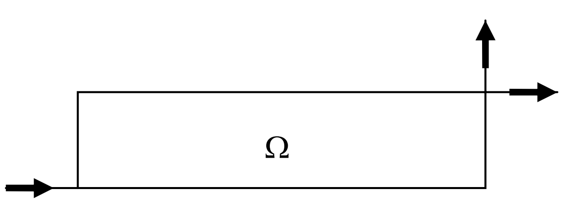
9. Bragg diffractions of a Bose-Einstein condensate (Slide 10 of Lecture 17)

A. Show that counter propagating laser beams with frequencies ω and ω+δ form a moving standing wave with velocity v. Determine v.

B. What is the resonance condition for a $|P=0>$ to $|P=2NP_{recoil}>$ transition. ($\theta=0$)

C. Compare Bragg diffraction of a BEC to that of a laser by a crystal.

10. Gyroscope based on a Mach-Zehnder interferometer



A. For light waves, show that the sensitivity of the interferometer to rotation (Sagnac effect) dΦ/dΩ is proportional to the enclosed area A. Here Φ is the phase difference between the two optical paths and Ω is the angular velocity.

B. Given A=(10cm)^2, what is the resolution of Φ you need to detect the spinning and the orbital revolution of the Earth?

C. Now consider matter waves. Show that $\phi = \frac{2m}{\hbar}\Omega \cdot A$.

(Hint: phase of a free atom evolves according to $\phi = \int kdx - \omega dt$, where $\hbar k$ and $\hbar\omega$ are the associated classical momentum and energy.)

D. Do you expect the sensitivity here is higher than in B under the same condition?