Quantum Optics: Atom-photon interaction: P452 Lec. 10 QO

Engineer interesting quantum states of atoms & photons

Atom-photon interaction

atom	photon	
classical	classical	Lorentz model
quantum	classical	Optical Bloch eqn
quantum	quantum	Dressed atom picture
		Jayne-Cummins model
		(Quantum Electro Dynamics)

Lorentz Model:

$+e$ $\overset{e^-}{\nearrow}$ x

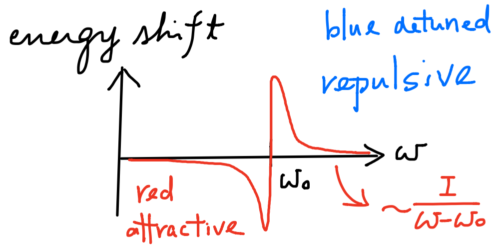
$$m\ddot{x} + 2\beta m\dot{x} + m\omega_0^2 x = -eE\cos\omega t, \quad (e>0)$$

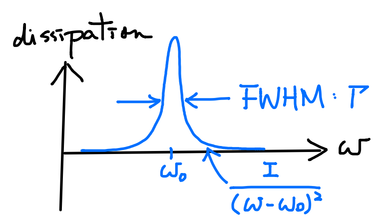
$$\text{Polarization } P = -ex \equiv \alpha E = -\frac{e}{m}\frac{1}{\omega^2-\omega_0^2+2i\beta\omega}E$$

In phase response: $\mathrm{Re}\,\alpha = -\frac{e}{m}\frac{\omega^2-\omega_0^2}{(\omega^2-\omega_0^2)^2+4\beta^2\omega^2} \Rightarrow$ energy shift $-p\cdot E$

$$\Rightarrow \text{light shift} \propto \frac{1}{\omega^2-\omega_0^2+4\beta^2\omega^2/(\omega^2-\omega_0^2)}I$$

Out of phase response: $\mathrm{Im}\,\alpha = \frac{e}{m}\frac{2\beta\omega}{(\omega^2-\omega_0^2)^2+4\beta^2\omega^2}I$





energy shift $E = \frac{\hbar\Gamma}{8}\frac{I/I_s}{\Delta/\Gamma}$ scattering rate (1/s): $s = \frac{\Gamma}{2}\frac{I/I_s}{1+I/I_s+4\Delta^2/\Gamma^2}$

energy dissipation $= s\hbar\omega$

Now level 2: quantum atoms + classical field:

$$H = \begin{bmatrix} \hbar\omega_0/2 & 0 \\ 0 & -\hbar\omega_0/2 \end{bmatrix} + \begin{bmatrix} 0 & -d\cdot E(t) \\ -d\cdot E^*(t) & 0 \end{bmatrix}, \quad E = E_0\cos\omega t$$

No general solution unless we are close to resonance $\omega \simeq \omega_0$

Rotation Wave app. (RWA) $\Rightarrow$ $H = \frac{\hbar\omega_0}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - \frac{\hbar\omega}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \frac{\hbar\Omega}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \frac{\hbar}{2}\begin{bmatrix} -\Delta & \Omega \\ \Omega & \Delta \end{bmatrix} = \frac{\hbar}{2}\left[-(\omega-\omega_0)\sigma_z + \Omega\sigma_x\right]$$

the state is $|\,\rangle = c_1|1\rangle + c_2|2\rangle$. excited population $|c_1|^2 = n_1$ $n_1+n_2=1$

ground population $|c_2|^2 = n_2$

If we only care about population $\rightarrow \rho$

Density matrix $\hat{\rho} = |\,\rangle\langle\,| = \sum_{ij}\rho_{ij}|i\rangle\langle j|$

$$= |c_1|^2|1\rangle\langle 1| + |c_2|^2|2\rangle\langle 2| + c_1c_2^*|1\rangle\langle 2| + c_1^*c_2|2\rangle\langle 1|$$

$i\hbar\partial_t\rho = [H,\rho] \Rightarrow \dot{\rho}_{11} = -i\Omega(\rho_{21}-\rho_{12}) - \Gamma\rho_{11}$

$\dot{\rho}_{22} = -\dot{\rho}_{11} + \Gamma\rho_{11}$ add by hand

$\dot{\rho}_{12} = -i\Omega(\rho_{22}-\rho_{11}) + i\Delta\rho_{12}$ saturation parameter $I/I_s = 2\Omega^2/\Gamma^2$

$\dot{\rho}_{21} = i\Omega(\rho_{22}-\rho_{11}) - i\Delta\rho_{12}$ $P \equiv \frac{I/I_s}{1+4\Delta^2/\Gamma^2}$

Steady state: $\Gamma\rho_{11} = -i\Omega(\rho_{21}-\rho_{12})$
$\Delta\rho_{12} = \Omega(\rho_{22}-\rho_{11})$ $\Rightarrow \rho_{11} = \frac{\Omega^2/\Gamma^2}{1+4\Delta^2/\Gamma^2+2\Omega^2/\Gamma^2} \equiv \frac{1}{2}\frac{P}{1+P}$

Scattering rate: $\Gamma\rho_{11} \equiv S = \frac{\Gamma}{2}\frac{P}{1+P}$

All coherence is assumed lost at this point. :(Next class :)

Compare classical & quantum treatments: