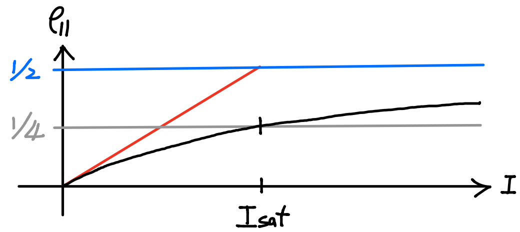
2017P452
Q&Q Lec11
11/6 C.C.

question from Frank: What is the meaning of saturation intensity?

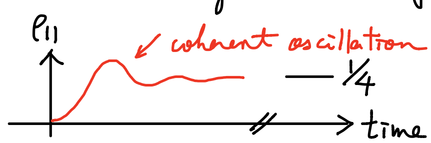
$$\rho_{11} = \frac{1}{2}\frac{P}{1+P} = \frac{1}{2}\frac{I/I_s}{1+I/I_s+4\Delta^2/\Gamma^2},$$ consider resonance scattering $\Delta = 0$



@ $I = I_{sat}$, you expect to reach the maximum of $\rho_{11} = 1/2$, but you only get $\rho_{11} = 1/4$.
I_{sat} is the intensity where you are off from the linear dependence by 2.

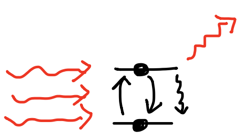
Thus far we talked about stationary solution, dynamics is surely more interesting.

Say at $I = I_{sat}$.

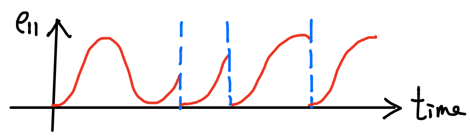


Why does it damp out?

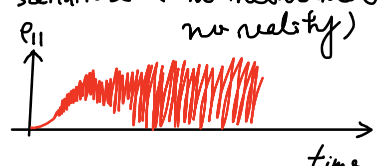
If you follow a single atom closely, does it damp out?



scenario 1 (hidden variable)

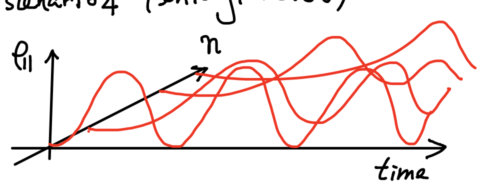


scenario 2 (no measurement no reality)



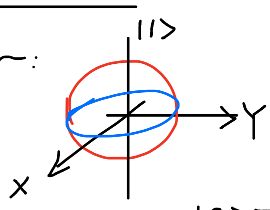
scenario 3 (multi-world)

scenario 4 (entanglement)



Come up with a good test, your name will be in textbooks if not getting the prize.

Bloch vector:



$|0\rangle = |g\rangle = |2\rangle$

Rules: 100% ground state ↓
100% excited state ↑
} no point to discuss coherence here

x-axis: $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

y-axis: $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

in general: $\cos\theta\,|0\rangle + e^{i\phi}\sin\theta\,|1\rangle$

This is a mapping from $SU(2)$ to $O(3)$ with the caveat that 2π rotation gives you (-1) sign

The vector rotates according to $\cos\theta|0\rangle + e^{i\phi}\sin\theta\, e^{i\Delta t}|1\rangle$. Δ = detuning from laser field.

The vector would be stationary if $\Delta = 0$ (laser on resonance)

When laser is on, it applies a torque to rotate the vector according to $\dot{\vec{v}} = \vec{\omega}\times\vec{v}$ where $\vec{v}$ is the vector & $\vec{\omega} = (\Omega, 0, -\Delta)$

Damping reduces the length of the vector : $\dot{\vec{v}} = \vec{\omega}\times\vec{v} - \Gamma\vec{v}$

Theoretical construction of Bloch vector $\vec{v}$

$\vec{v} = \vec{\sigma}$ ← Pauli matrices

$\langle\vec{v}\rangle = \mathrm{tr}[\hat{\sigma}\hat{\rho}]$ $\Rightarrow$ $v_x = \langle\sigma_x\rangle = \mathrm{tr}\left[\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}\right] = \rho_{12}+\rho_{21}$

$v_y = \langle\sigma_y\rangle = i(\rho_{12}-\rho_{21})$

$v_z = \langle\sigma_z\rangle = \rho_{11}-\rho_{22}$

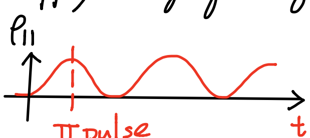
$$\partial_t\vec{v} = \frac{1}{i\hbar}[\vec{v}, H] = \frac{1}{2i}[\vec{\sigma}, \omega_0\sigma_z - \Delta\omega_z + \Omega\omega_x]$$

$$= \vec{\omega}\times\vec{v}, \text{ where } \vec{\omega} = (\Omega, 0, -\Delta)$$

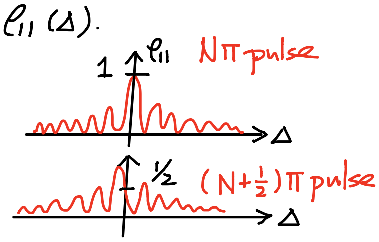
Application: Precision Measurements and quantum information

Rabi Spectroscopy

Apply driving field for a duration t and measure $\rho_{11}(\Delta)$.

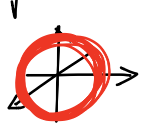


$\rho_{11} \sim \sin^2\Omega_g t$ $\quad\Omega_g = \sqrt{\Omega^2+\Delta^2}$

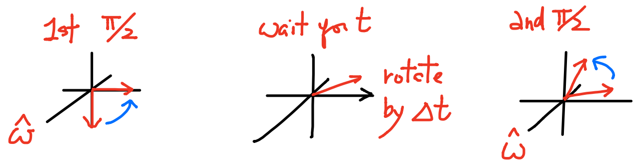


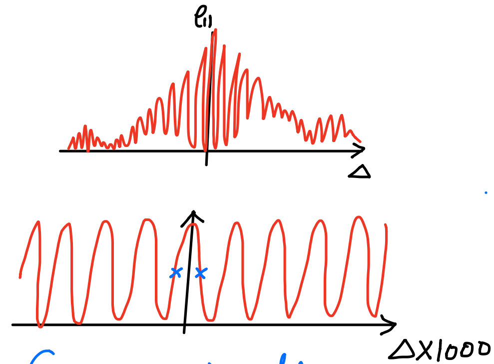
Signal extremely sensitive to energy offset Δ.

Bloch sphere representation: rotation around x-axis.



Ramsey spectroscopy: $\pi/2$ pulse $-\ t\ -$ $\pi/2$ pulse



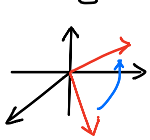


Compare signals on both sides of the central fringe.

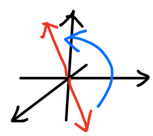
Precision of Rabi & Ramsey spectroscopies

$$\Delta\omega = C \underbrace{\frac{1}{t}}_{\text{uncertainty for a single atom}} \underbrace{\frac{1}{\sqrt{N}}}_{\text{shot noise limit from independent events/atoms}}$$

Quantum gates on single qubit:



$\pi/2$ pulse along x



π pulse along x

$\hat{U}_H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \equiv$ —$\boxed{H}$—

Hadamard gate

$U_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \equiv$ —$\oplus$—

NOT gate