Light shift:

Now we will quantize both atomic energy and radiation field.

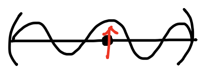
atom: $\frac{\hbar}{2}\omega\sigma_z$

photons: $\hbar\omega(\hat{a}^+\hat{a}+\frac{1}{2})$

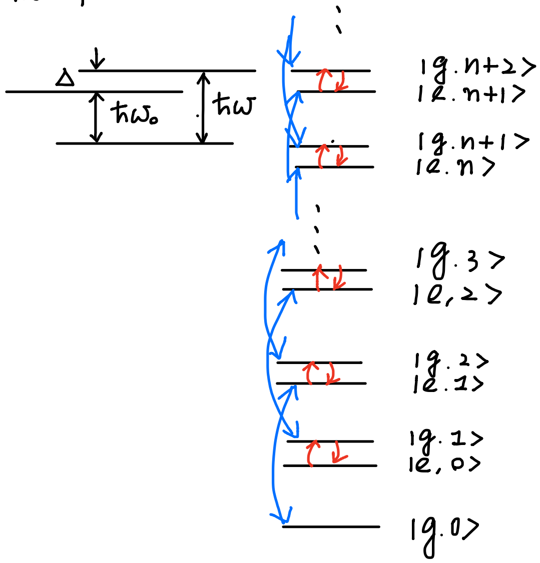
atom-photon coupling $\quad g\,\overbrace{(\sigma_+ + \sigma_-)}^{x}\overbrace{(\hat{a}^+ + \hat{a})}^{\cos\omega t} \underset{RWA}{\approx} g\,(\sigma_+\hat{a} + \sigma_-\hat{a}^+)$

} Jaynes-Cummings model

state $|\ \rangle = |\sigma\rangle \otimes |n\rangle \quad \sigma = 0$ or 1. $\quad n = 0.1.2.\dots$

model: 

no interaction:



assume $\Delta = \omega - \omega_0 > 0$

RW coupling $\quad \sigma^+ a + \sigma^- a^+$

CRW coupling $\quad \sigma a + \sigma^+ a^+$

If we ignore CRW terms, all couplings occures between $|g, n+1\rangle$ and $|e,n\rangle$

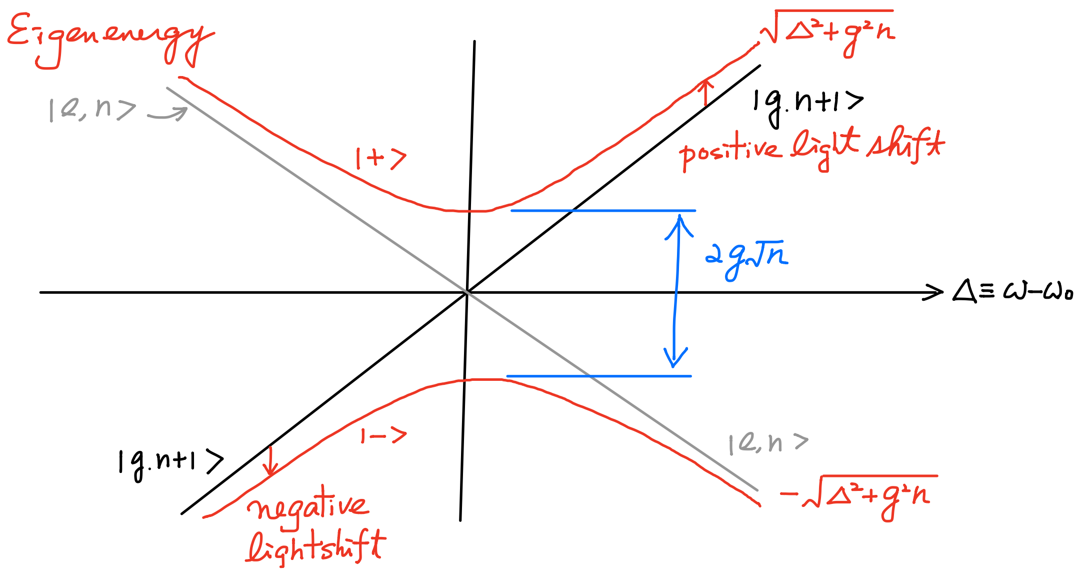
consider only a pair of coupled states: $|\ \rangle = \begin{pmatrix} |e,n\rangle \\ |g,n+1\rangle \end{pmatrix}$

$$H = \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\omega(b^+b+\frac{1}{2}) + g\sqrt{n}\,(\sigma^+ b + \sigma b^+)$$

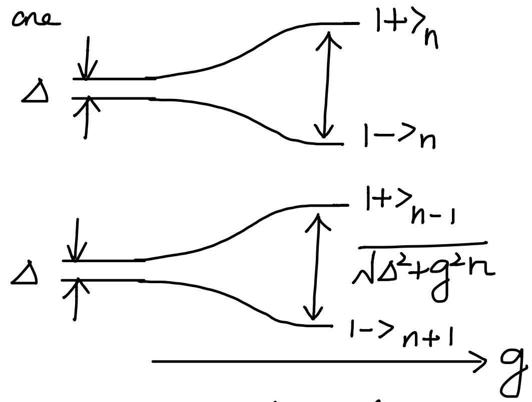
$$\langle H\rangle = \frac{1}{2}\hbar\omega_0\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix} + \hbar\omega\begin{pmatrix} n+\frac{1}{2} & 0 \\ 0 & n+\frac{3}{2}\end{pmatrix} + g\sqrt{n}\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix} = -\frac{1}{2}\hbar\Delta\,\sigma_z + g\sqrt{n}\,\sigma_x + \text{const}\,\hat{1}$$

We have seen this a couple of times: Feshbach, Rabi flopping .....

This is generally how 2 modes are coupled.



Thus the eigenstates are



Thus we have a new prediction for the light shift

1. First, there are only eigenstates, $|g\rangle$ & $|e\rangle$ are "dressed" by the radiation field. This is the dressed atom picture

2. Light shift is the work done to bring an atom into the beam

3. Shift $= \frac{1}{2}(E_+ - \Delta)$ for blue-detuned light $\Delta > 0$

$= \frac{1}{2}(E_- + \Delta)$ for red-detuned light $\Delta < 0$

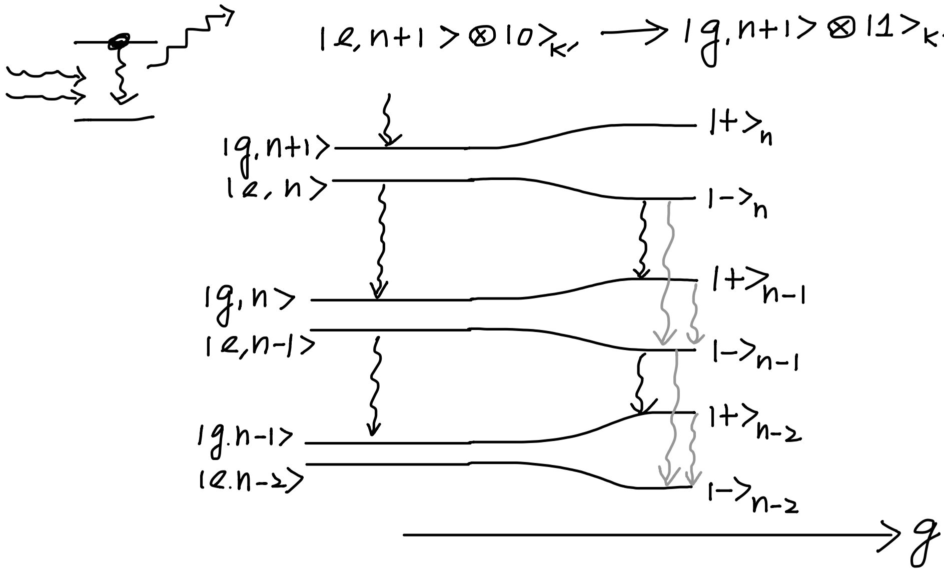
$$\Rightarrow \Delta E_L = \underline{\underline{\frac{1}{2}\left(\sqrt{\Delta^2+g^2 n} - \Delta\right)}} \xrightarrow[\Delta \gg g\sqrt{n}]{} \frac{1}{4}\,\frac{g^2 n}{\Delta}$$

a more accurate formula!!

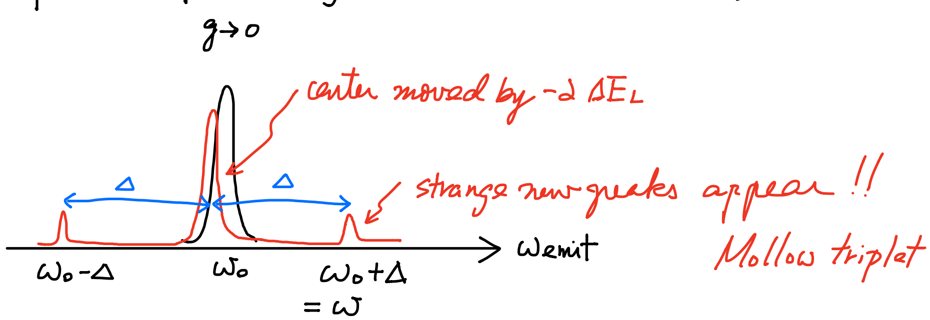
$\left(\text{compared to } \Delta E = \frac{\Gamma}{8}\,\frac{I/I_s}{\Delta/\Gamma}\right)$

Example 1. Can we include spontaneous emission?

$|e, n+1\rangle \otimes |0\rangle_{k'} \longrightarrow |g, n+1\rangle \otimes |1\rangle_{k'}$



Thus we predict spontaneously emitted photons should carry new components:



Now question, illuminating some atoms with radiation at ω, we actually get emission at $\omega_0 - 2\Delta E_L \approx \omega_0$ & $\omega_0 \pm \Delta = \omega,\ 2\omega_0 - \omega$.

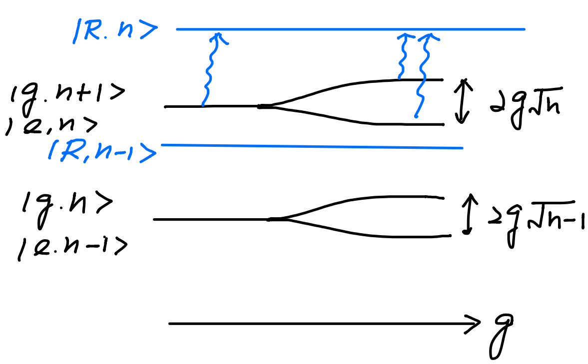
So if laser is detuned by $\Delta = -1\,GHz$, we see atoms emit photon at $\omega_0 + 1GHz = \omega + 2GHz$. We gain $2GHz$ of energy!?

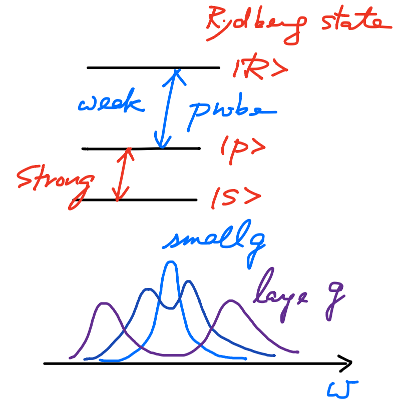
What's going on!?

Mostly, the emitted photons have the same freq as the incident photons: Rayleigh scattering. (apart from light shift & recoil.)

Example 2: Autler-Townes effect

If laser is exactly on resonance





Many more interesting stuffs from dressed atom picture.