2017 P452 Q&Q Lec. 14 11/13/2017 Cheng Chin

Entanglement

Definition: object A can be described by a set of states $|\Psi_A\rangle = \sum_i a_i |i\rangle$
object B by $|\Psi_B\rangle = \sum_j b_j |j\rangle$. A quantum state of the system A & B is entangled if it's not separable.

A separable state is $|\Psi_{AB}\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$

Note that all pure states can be written as $|\ \rangle = \sum_{ij} A_{ij} |i\rangle \otimes |j\rangle$

If the system is not a pure state, separable means

$$\rho_{AB} = \sum_k P_k \rho_A^k \otimes \rho_B^k \quad , \quad \sum_k P_k = 1$$

Math: $R^3 \oplus R^2 = R^5$ direct sum
$R^3 \otimes R^3 = R^6$ direct product
What's the difference?

Examples we talk about: 2-spinor: $|S=1, m_s = \pm 1, 0\rangle = |\pm\frac{1}{2}, \pm\frac{1}{2}\rangle, |\frac{1}{2}, -\frac{1}{2}\rangle + |\frac{1}{2}, -\frac{1}{2}\rangle$ (triplet)

$|S=0, m_s=0\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle - |-\frac{1}{2}, \frac{1}{2}\rangle$

singlet $= |\frac{1}{2}, -\frac{1}{2}; \hat{e}_n\rangle - |-\frac{1}{2}, \frac{1}{2}; \hat{e}_n\rangle$
($\hat{e}_n$: any basis; x, y, z ...)

Ideal Bose condensate $|BEC\rangle = |g\rangle \otimes |g\rangle \otimes \cdots$
(in lattice) $= (\frac{1}{\sqrt{N}} \sum_i \hat{a}_i^+)^N |0\rangle$

Ideal Mott insulator $|MI\rangle = \prod_i \hat{a}_i^+ |0\rangle$

Bell state basis $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$

$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$

GHZ (Greenberger, Horne, Zeilinger) state

$$\frac{1}{\sqrt{2}}(|111\rangle + |000\rangle)$$

Noon state $\frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle)$ a kind of cat state.

If entanglement merely suggests the observable of object influences the other one, is a bomb that breaks into 2 pieces going in opposit direction P and -P also creates entangled pieces?

What's the difference between classical & quantum entanglement?

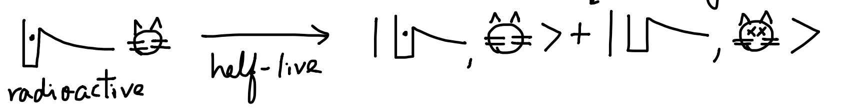
⇒ quantum entanglement must be phase cohent.

$\rho_{ij} = \begin{bmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{bmatrix}$ is not an entanged state!!

Unfortunately many papers confuse classical & quantum entanglement.

How about a spontaneously emitted photon from an stationary atom.

Is phase coherence preserved? EPR pair &



Does  more like particles that can entangle or ~~must be~~ classical?

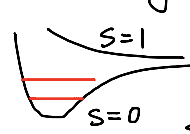
Is decoherence intrinsic for macroscopic object or just loss of information?

Does it make sense to talk about decoherence for a single event?

Does single event ever decohere? Complex systems must decohere?

Is entropy really increasing? Is every process fundamentally reversable?

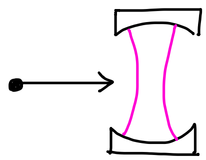
Finally, ain't most of the molecular states entangled states of atoms? What the fuss is about?



all H_2 are entangled spin states.

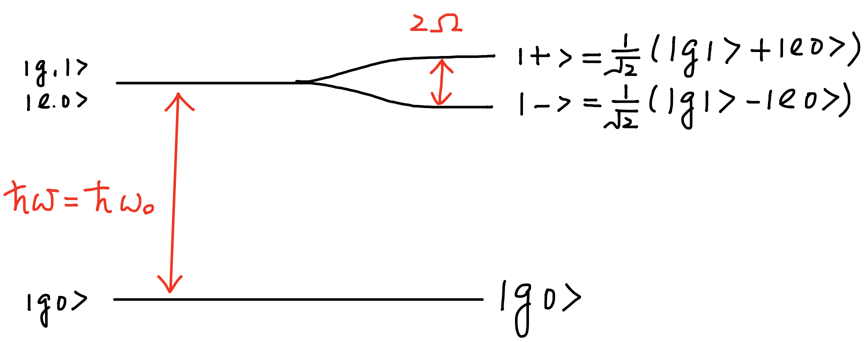
Creating entangled states:

$|\psi\rangle = |e/g\rangle \otimes |n\rangle$

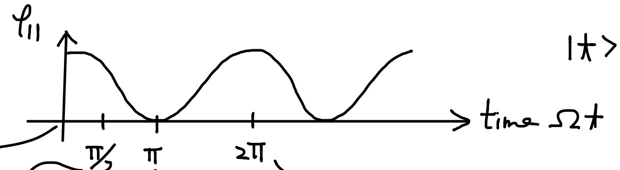


Assume we have an empty and resonant cavity to begin with.

$$H = \begin{bmatrix} \hbar\omega & \hbar\Omega/2 \\ \hbar\Omega/2 & \hbar\omega \end{bmatrix}$$



If we start with $|e,0\rangle$ & suddenly turn on coupling



$$|t\rangle = \begin{pmatrix} \cos\frac{\Omega}{2}t & \sin\frac{\Omega}{2}t \\ -\sin\frac{\Omega}{2}t & \cos\frac{\Omega}{2}t \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\|e0\rangle$	$\frac{1}{\sqrt{2}}(\|e0\rangle + \|g1\rangle)$	$\|g1\rangle$	$-\|e0\rangle$
$\|g1\rangle$	$\frac{1}{\sqrt{2}}(-\|e0\rangle + \|g1\rangle)$	$-\|e0\rangle$	$-\|g1\rangle$
	$\pi/2$ pulse	π-pulse	2π-pulse

$|e1\rangle + |g0\rangle$

We see immediately that a single $\pi/2$ pulse creates entangled state

Phase gate: atom in $|g\rangle$ or some other state $|i\rangle$. cavity in $|0\rangle$ or $|1\rangle$

Apply a 2π-pulse which flips the sign only when we have $|g1\rangle$

$$\Rightarrow U_\pi = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix}$$, which is equivalent to CNOT gate as we will see.

Finally GHZ state can be realize with 2 atoms + 1 cavity plus 2 pulses. See HW.