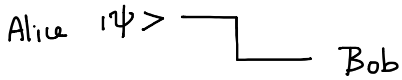
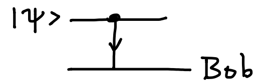
Quantum teleportation

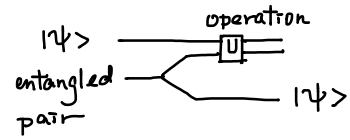
Transfer a quantum state ψ from A to B. A and B are far separated.



no physical transport



Quantum state cannot be copied
no-cloning theorem



is this possible?
no-communication theorem

Proof of no clone theorem:

Proof of no-communication theorem:

$$\rho_{AB} = \rho_A \otimes \rho_B \qquad \rho_A = |A\rangle\langle A|$$

Alice does some operation : $|A'\rangle = U_A|A\rangle \Rightarrow \rho_{A'} = U_A^* |A\rangle\langle A| U_A$

$$\rho_{A'B} = U_A^* \rho_A U_A \otimes \rho_B$$

$$\rho_{B'} = tr_A\, \rho_{A'B} = tr(U_A^* \rho_A U_A) \otimes \rho_B = tr(\rho_A U_A U_A^*) \otimes \rho_B = \rho_B$$

Nothing will happen to Bob.

This also implies no-clone theorem. If Bob can duplicate many qubits, he would know whether a measurement have been made by Alice

If Alice measures $|\uparrow\rangle$. Bob gets $|\downarrow\rangle |\downarrow\rangle |\downarrow\rangle |\downarrow\rangle$ by duplication.
If Alice does not measure Bob gets random result.

So the best people have figured out to transfer a quantum state is BBCJPW protocol: PRL 70, 1895 (1993)

Step 1. A & B share a pair of entangled spins $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle_{AB} - |\downarrow\uparrow\rangle_{AB})$

Step 2. Alice projects her spin and the information into Bell state basis and collapse the wavefunction to one of them.

$$|\uparrow\psi\rangle - |\downarrow\psi\rangle = A(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) + B(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) + C(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) + D(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$$

Say. $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle \Rightarrow \text{LHS} = \alpha|\uparrow\uparrow\rangle + \beta|\uparrow\downarrow\rangle - \alpha|\downarrow\uparrow\rangle - \beta|\downarrow\downarrow\rangle$

$\Rightarrow \alpha = C + D. \quad -\beta = C - D$

$\Rightarrow C = \frac{\alpha - \beta}{2}, \quad D = \frac{\alpha + \beta}{2}$

Alice's measurement projects Bob's spin

case A: Bob gets $\hat{\sigma}_z|\psi\rangle$

case B: $|\psi\rangle$

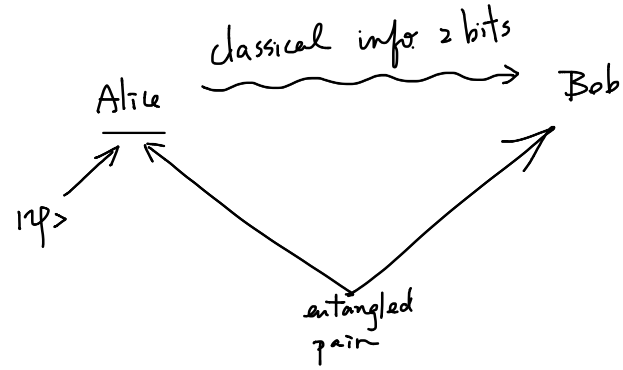
since AB anti-corr. AA' anti-corr. ⇒ BA' correlates

case C: $\hat{\sigma}_x\hat{\sigma}_z|\psi\rangle$

case D: $\hat{\sigma}_x|\psi\rangle$

AB anti-corr. AA' correlate ⇒ BA' anti-correlate.

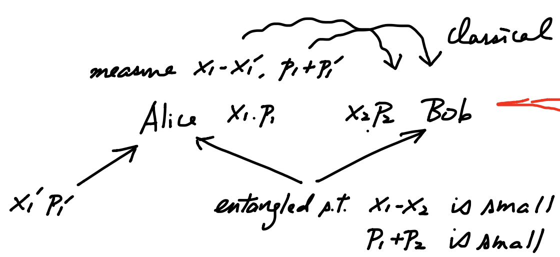
Step 3: Alice tells Bob which case she got & Bob can determine $|\psi\rangle$



Remaining questions: How to perform a measurement in Bell state basis?

Is this the best we can do?

Teleport a continuous quantum variable $\psi(x)$?



Let me think...

$x_2 - (x_1 - x_1') = x_1' - (x_1 - x_2) \approx x_1'$

$p_2 + p_1 + p_1' = p_1' + p_1 + p_2 \approx p_1'$

so I offset my x_2 by $(x_1 - x_1')$ and I offset my p_2 by $-(p_1 + p_2)$ then I am golden! ($x_2 \approx x_1'$, $p_2 \approx p_1'$)