Lecture 16 Cavity QED

1. No communication proof.

$\rho_{AB} = \sum_i \rho_A^i \otimes \rho_B^i$

Operation on A : $|A\rangle_i \to |A\rangle_i' \Rightarrow \rho_A^i = |A\rangle_i \langle A|_i \longrightarrow \rho_A^{i\prime} = |A\rangle_i' \langle A|_i' = U_A^\dagger \rho_A^i U_A$

$\Rightarrow \rho_{AB}' = \sum_i U_A^\dagger \rho_A^i U_A \otimes \rho_B^i$

$\Rightarrow \rho_B' = tr_A\, \rho_{AB}' = \sum_i tr_A (U_A^\dagger \rho_A^i U_A) \otimes \rho_B^i = \sum_i tr_A (\cancelto{1}{U_A U_A^\dagger} \rho_A^i) \otimes \rho_B^i = \rho_B.$

Thus operation on B does not change the statistics of A.

Cavity QED:

We have discussed Jaynes-Cummings model: (RWA)

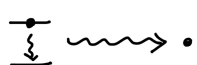
$$H = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega(\hat{a}^\dagger a + \tfrac{1}{2}) + g(\sigma_+ \hat{a} + \sigma_- \hat{a})$$

and discussed it's eigenenergy, light shift, emission spectra...

spontaneous emission was introduced semi-classically

New things to discuss here: Spontaneous emission Γ & cavity decay K.

Spontaneous emission is how fast an excited atom couples to vacuum



The rate ranges from 1/ns in optical domain to < 1/s μwave domain & forbidden transitions.

Why do they vary so much? Can we describe it with J.C. model?

Consider a single excited atom in a large box L^3.

$$-d\cdot E_v \qquad H_{ap} = \sum_K g_K (\sigma_+ \hat{a}_K + \sigma_- \hat{a}_K^\dagger)$$

$\frac{1}{2}\epsilon_0 E_v^2 V = \frac{1}{4}\hbar\omega$

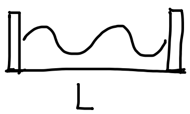
$\Rightarrow E_v = \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}$

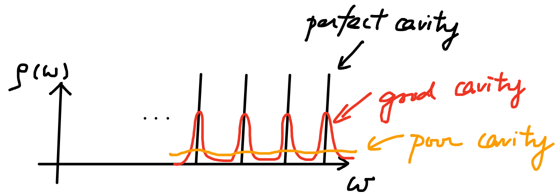
Transition rate $\Gamma = \frac{2\pi}{\hbar}|\langle f|H|i\rangle|^2 \rho$ ← D.O.S.

$$\Rightarrow \Gamma = \frac{2\pi}{\hbar} g_k^2 \frac{V}{\pi^2}\frac{\omega^2}{c^3} = \frac{V}{\pi^2}\frac{\omega^2}{c^3}\frac{2\pi}{\hbar} d_x^2 \frac{\hbar\omega}{2\epsilon_0 V}. \qquad d_x^2 = \frac{d^2}{3}$$

$$= 2\pi \frac{V\omega^2}{\pi^2 c^3}\frac{d^2}{3}\frac{\omega}{2\hbar\epsilon_0 V}$$

Now let's consider a finite size cavity. (good cavity: 2 mirrors. bad cavity vacuum chamber)





$N\lambda/2 = L$

$\Rightarrow \omega = \frac{c}{\lambda} = N \frac{c}{2L}$

1/ round trip time $2L/c$

$$\rho(\omega) = \frac{\kappa}{2\pi}\frac{1}{(\omega-\omega_c)^2 + (\kappa/2)^2}, \quad \kappa = \frac{\omega_c}{Q}$$

← quality factor.

Right on resonance $\omega_c = \omega_0$

$$\rho(\omega_0) = \frac{2}{\pi}\frac{1}{\kappa} = \frac{2}{\pi}\frac{Q}{\omega_c}$$

Compare the α:

Free space: $\Gamma_f = 2\pi \frac{V\omega^2}{\pi^2 c^3}\frac{d^2}{3}\frac{\omega}{2\hbar\epsilon_0 V}$

Cavity: $\Gamma_c = 2\pi \frac{2}{\pi}\frac{Q}{\omega_c}\mu^2 \frac{\omega_c}{2\hbar\epsilon_0 V}$

$\Rightarrow$ Purcell factor $\frac{\Gamma_c}{\Gamma_f} = \frac{3}{4\pi^2} Q \frac{\lambda^3}{V}$

For a perfect cavity, spontaneous emission can be inhibited or enhanced by the cavity!!

Decay of atoms and cavity are two factors that limits the coherent evolution of cavity QED.

Total Hamiltonian
$$H = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega_c(a^\dagger a + \tfrac{1}{2}) + g(\sigma^+ a + \sigma^- a^\dagger)$$
$$+ g\sum_i(\sigma^+ b_i + \sigma b_i^\dagger) + \sum_i \frac{\hbar\omega_i}{2}(b_i^\dagger b_i + \tfrac{1}{2})$$
$$+ \kappa\sum_j(a^\dagger c_j + a c_j^\dagger) + \sum_j \frac{\hbar\omega_j}{2}(c_j^\dagger c_j + \tfrac{1}{2})$$