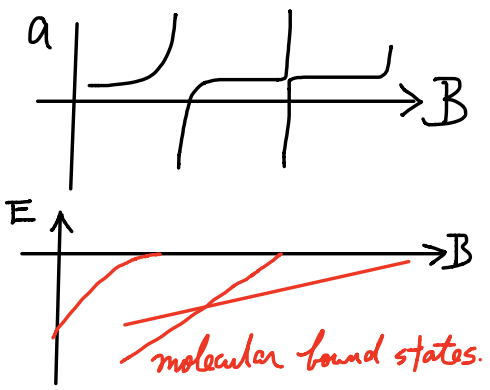
Feshbach Resonance



Features

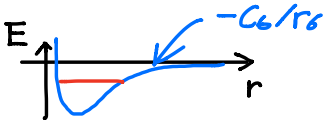
1. field dependent
2. Strong tunability
3. Associated with molecular state.
4. universal dependence

$$a = a_{bg}\left(1 - \frac{\Delta}{B - B_0}\right)$$

More on atomic collisions :

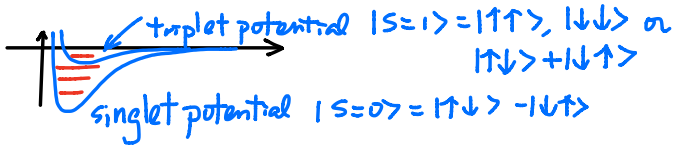
Molecular potential : single channel vs. multichannel scattering.

Structureless atoms: He. Ar...



Alkali-like ctoms: H. Li. Rb. Cs

same $-C_6/r_6$ for both potentials



Questions: Why is singlet deeper than triplet?

More details: hyperfine structure and Zeeman shift.

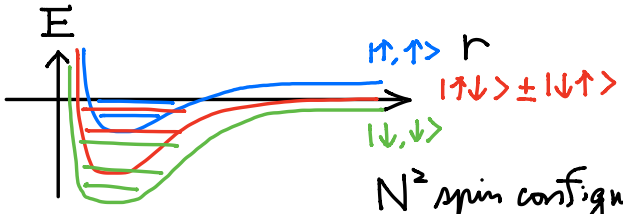
$$H = \underbrace{-C_6/r^6}_{\text{scalar shift}} + \underbrace{E_{ex}(S_1 \cdot S_2)}_{\text{exchange}} + \underbrace{A\, S_1 \cdot i_1 + A\, S_2 \cdot i_2}_{\text{hyperfine}} - \underbrace{\mu_1 \cdot B - \mu_2 B}_{\text{Zeeman effect.}}$$

exchange: separate singlet & triplet

hyperfine: mix e & nuclear spin

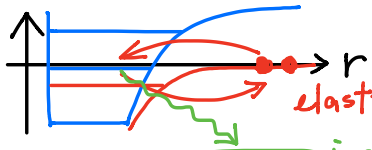
Zeeman effect: magnetic field tunability.

Simplified model:



N^2 spin configurations
scattering channels.

Zoom in



elastic collision

inelastic, spin flip
exothermic collision
to lower channel.

Scattering wavefunction

$$\vec{\psi}(r) = \frac{1}{r} e^{-ikr}\vec{\chi}_{in} - \frac{1}{r} e^{ikr}\hat{S}\hat{\chi}_{in}$$

$$= \frac{1}{r}\left[e^{-ikr} - \hat{S}e^{ikr}\right]\hat{\chi}_{in}$$

Examples: no collision: $\hat{S} = \hat{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

only elastic collision: $S_{ij} = \delta_{ij} e^{2i\theta_i}$

cross-section $\sigma = \int |f|^2 \, d\Omega$, scattering amp $f = \frac{S-1}{2ik}$

$$= \frac{4\pi}{k^2}\sin^2\theta$$

elastic collision cross-section of the ith channel: $\sigma_i = \frac{4\pi}{k^2}|S_{ii} - 1|^2$ *

inelastic collision cross section from $i \to j$: $\sigma_{ji} = \frac{4\pi}{k^2}(1 - |S_{ij}|^2)$

Probability conservation: $\sum_j |S_{ij}|^2 = 1$

For collisions between bosons, wavefunc is sym. $\Rightarrow \sigma_{el} = \frac{8\pi}{k^2}\sin^2\theta$

if we use $k\cot\theta = -\frac{1}{a}$ at low k $\Rightarrow \sigma = \frac{8\pi a^2}{1 + k^2 a^2}$

For identical Fermions, wavefunction cannot be symmetrized

$\Rightarrow \sigma = 0$.