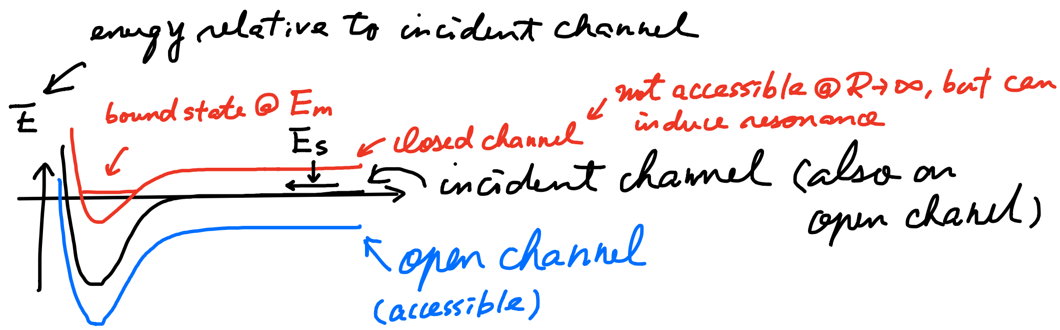
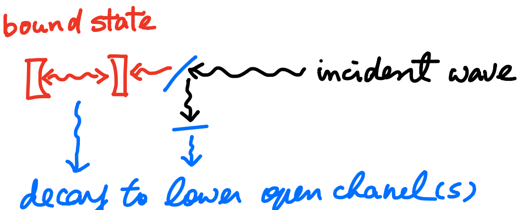
Feshbach resonance



Resonance occurs when scattering energy E_s is near that of a bound state E_m in a closed channel.

A quantum optics analog:



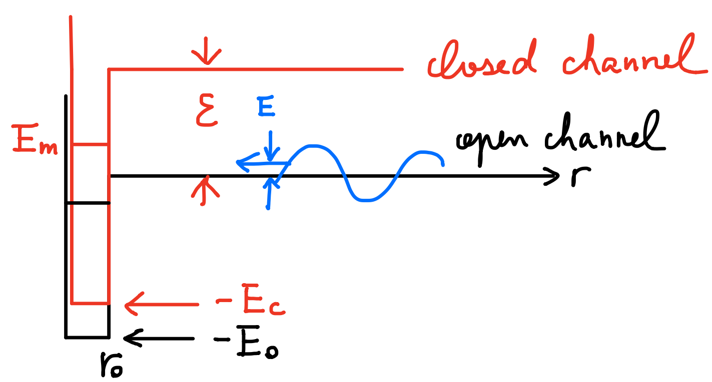
Enhanced loss: inelastic Feshbach resonance

Enhanced elastic collisions: elastic Feshbach resonance

We will only discuss elastic Feshbach resonance here

You can write a paper to generalize what we do here to include inelastic resonance.

Model:



$$\hat{H}\varphi = E\varphi . \quad \varphi = \begin{bmatrix} \varphi_c(r) \\ \varphi_o(r) \end{bmatrix} \quad V(r>r_0) = \begin{bmatrix} \varepsilon \xrightarrow{} \infty & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \left[-\frac{\hbar^2}{m}\nabla^2 + \hat{V}(r)\right]\vec{\varphi} = E\vec{\varphi} \qquad V(0<r<r_0) = -\begin{bmatrix} E_c & V \\ V & E_0 \end{bmatrix}$$

E_c : closed channel depth $\sim 1000\,K$
E_0 : open channel depth $\sim 1000\,K$

$20\,GHz = 1\,K$

V: mixing between the two channels $\sim$ hyperfine $\sim 1\,K$

Scattering energy $E \sim \mu K$

ε : closed channel energy $\sim$ hyperfine $\sim 1\,K \to \infty$

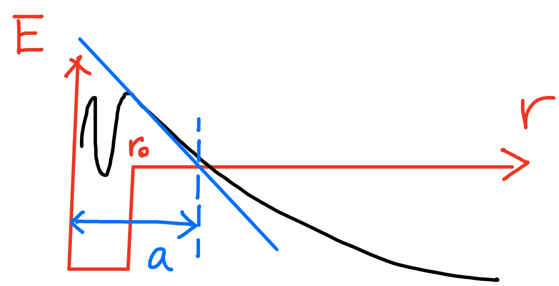
$\Rightarrow$ outside the potential : $r\varphi(r>r_0) = \begin{pmatrix} 0 \\ \sin(kr+\delta) \end{pmatrix} = \sin(kr+\delta)|0\rangle$

inside the potential : $r\varphi(r<r_0) = A_+ \sin k_+ r\, |+\rangle + A_- \sin k_- r\, |-\rangle$

$|\pm\rangle$ are the eigenstates.

$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |0\rangle \\ |c\rangle \end{pmatrix}$$

B.C. $\varphi(r_0^-) = \varphi(r_0^+)$
$\varphi'(r_0^-) = \varphi'(r_0^+)$



$\Rightarrow \varphi_c(r_0^+) = \varphi_c(r_0^-) = 0$

$$\left.\frac{r\varphi'}{r\varphi}\right|_{r_0^+} = k\cot(kr_0+\delta)$$

$$\left.\frac{r\varphi'}{r\varphi}\right|_{r_0^-} = k_+\cot k_+r_0 \cos^2\theta + k_-\cot k_-r_0 \sin^2\theta$$

Remember in the $k\to 0$ limit the intercept is $r=a$.

$\Rightarrow \lim_{k\to 0} k\cot(kr_0+\delta) = \frac{1}{r_0-a}$

$$\Rightarrow k\cot(kr_0+\delta) = k_+\cos^2\theta \cot k_+r_0 + k_-\sin^2\theta \cot k_-r_0$$

in the limit of small θ, 2nd term $\to 0$ & $|+\rangle \simeq |0\rangle$

We recover single channel result (HW2)

$k\cot(kr_0+\delta_{bg}) = k_+\cot k_+r_0$, thus we may rewrite

$$k\cot(kr_0+\delta) = k\cot(kr_0+\delta_{bg}) + k_-\sin^2\theta\cot k_- r_0$$

In the limit of $k\to 0$. we have

$$\frac{1}{r_0-a} = \frac{1}{r_0-a_{bg}} + k_-\theta^2\cot k_- r_0$$

$$\approx -\frac{1}{r_0}\frac{\Gamma/2}{E_m}$$

HW3 you will see this term is only important when there is a bound state near by.

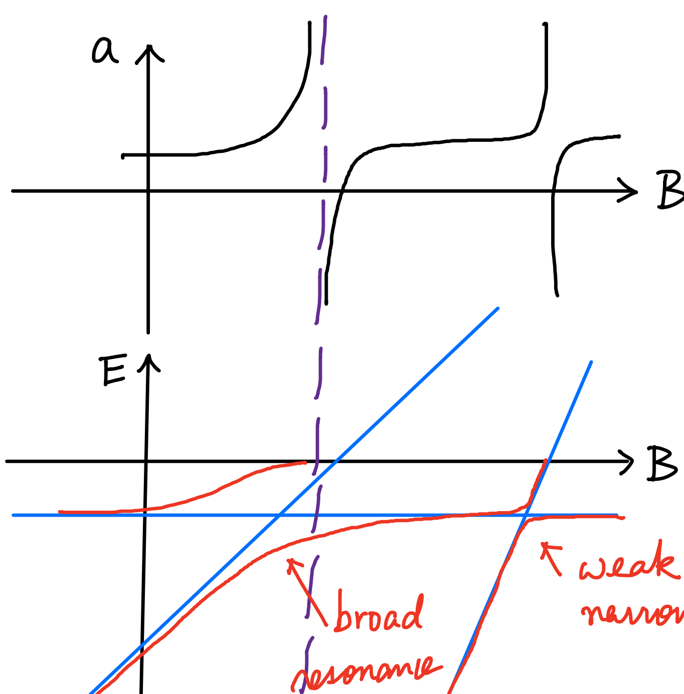
$$\Rightarrow a = a_{bg} - \frac{r_0\Gamma'}{E_m+\Delta E}$$

$\leftarrow$ coupling strength

$\nwarrow$ self energy of the mol.

To show the magnetic tunability, we assume E_m can be tuned as $E_m(B) = E_m + \mu_m B$

Show that (HW3) we arrive at $a = a_{bg}\left(1-\frac{\Delta B}{B-B_0}\right)$



blue: bare molecular states

red: eigenstates.