1A: Short range: $\left[k^2\hat{1} - \begin{pmatrix} \xi_c^2 & \epsilon \\ \epsilon & \xi_o^2 \end{pmatrix}\right]\begin{pmatrix}\varphi_c\\ \varphi_o\end{pmatrix} = \xi_\pm^2 \begin{pmatrix}\varphi_c\\ \varphi_o\end{pmatrix}$

diagonalize $\Rightarrow$ $\xi_\pm^2 = \frac{1}{2}\left\{\xi_c^2 + \xi_o^2 \pm \sqrt{(\xi_c^2-\xi_o^2)^2 + 4\epsilon^2}\right]$

$r > r_0$: $r\varphi = \sin(kr+\delta)\,|0\rangle$

$r < r_0$: $r\varphi = A_+ \sin \xi_+ r + A_- \sin \xi_- r$

matching B.C. $\varphi'/\varphi\,|_{r_0^-} = \varphi'/\varphi\,|_{r_0^+}$ gives

$$k\cot(kr_0+\delta) = k_+ \cos^2\alpha \cot k_+ r_0 + k_- \sin^2\alpha \cot k_- r_0$$

1B. $(P^2/2\mu - \hbar^2\xi_c^2/2\mu)\, r\varphi = E_c r\varphi$ & $\varphi_c(r=r_0)=0$

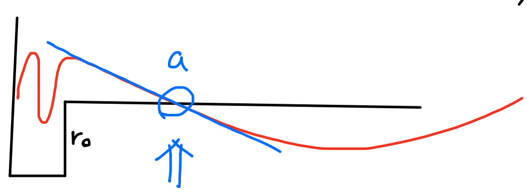
$\Rightarrow \sin\sqrt{\xi_c^2 + 2\mu E_c/\hbar^2}\; r_0 = 0$

$\Rightarrow \sin(\xi_c r_0 + \mu E_c r_0/\hbar^2\xi_c) = 0$

for $E_c \ll \hbar^2\xi_c^2/2\mu$. expanding the above gives $\cot \xi_c r_0 = -\cot \frac{\mu E_c r_0}{\hbar^2 \xi_c} \approx -\frac{\hbar^2\xi_c}{\mu E_c r_0}$

$\Rightarrow \xi_- \sin^2\alpha \cot \xi_- r_0 \approx -\frac{\Gamma/2}{r_0 E_c}$ & $\Gamma \approx 4\alpha^2 \frac{\hbar^2\xi_c^2}{2\mu}$

1C



Zero crossing defines a, we can use the B.C. $\varphi(r_0)$ & $\varphi'(r_0)$ to get a.

Matching $r\varphi = A(r-a)$ with $\lim_{k\to 0} \sin(kr+\delta)$ @ $r=r_0$

$$\Rightarrow \frac{(r\varphi)'}{r\varphi}\Big|_{r=r_0} = k\cot(kr_0+\delta) = -\frac{1}{a-r_0}$$

Thus $-\frac{1}{a-r_0} = \xi_+ \cos^2\alpha \cot \xi_+ r_0 - \frac{\Gamma/2}{r_0 E_c}$

We can see away from resonance $E_c \to \infty$, only the 1st term is left

Thus it can be identified as the background term $-\frac{1}{a_{bg}-r_0}$

$$\Rightarrow \frac{1}{a-r_0} = \frac{1}{a_{bg}-r_0} + \frac{\Gamma/2}{r_0 E_c}$$

1D. Use $E_c = \mu(B-B_c)$, we can rearange and solve $a(B)$, which gives

$$a = a_{bg}\left(1 - \frac{\frac{\Gamma}{2\mu}\frac{(r_0-a_{bg})^2}{r_0 a_{bg}}}{B - B_c + \frac{\Gamma(a_{bg}-r_0)}{2\mu r_0}}\right) \Rightarrow \Delta B = \frac{\Gamma(a_{bg}-r_0)^2}{2r_0\mu r_0 a_{bg}}.\quad B_0 = B_c - \frac{\Gamma(a_{bg}-r_0)}{2\mu r_0}$$

1E. When $E_c = 0$, $a = r_0$. When $a = \pm\infty$, $E_c = \frac{\Gamma}{2}\left(1 - \frac{a_{bg}}{r_0}\right)$

2A $\nabla^2 r\varphi_0 = k_m^2 r\varphi_0 \Rightarrow r\varphi = \binom{0}{e^{-k_m r}}$

2B L.H.S. $\frac{(r\varphi)'}{r\varphi} = -k_m$.
R.H.S. $E_c \to E_c + E_b$

$$\left.\right\} k_m = \frac{1}{a_{bg} - r_0} + \frac{\Gamma/2}{r_0(E_b + E_c)}$$

2C. $k_m \to 0 \Rightarrow E_b \to 0 \Rightarrow$ $0 = \frac{1}{a_{bg} - r_0} + \frac{\Gamma/2}{r_0 E_c}$
Compared to $\frac{1}{a - r_0} = \frac{1}{a_{bg} - r_0} + \frac{\Gamma/2}{r_0 E_c}$ $\left.\right\} \Rightarrow a = \pm\infty$

2D. For small bonding energy $E_b \ll \frac{\hbar^2}{2\mu r_0^2}$. $k_m r_0 \ll 1$. we rewrite

$$k_m = \frac{1}{a_{bg} - r_0} + \frac{\Gamma/2}{r_0(E_c + \hbar^2 k_m^2/2\mu)} \quad \text{and use}$$

$\frac{1}{a - r_0} = \frac{1}{a_{bg} - r_0} + \frac{\Gamma/2}{r_0 E_c}$ to eliminate E_c. Expand k_m to 2nd order in a gives $E_b = \frac{\hbar^2 k_m^2}{2\mu} = \frac{\hbar^2}{2\mu A^2} + O(a^{-4})$. where $A = r_0 - \frac{\hbar^2}{2\mu a_{bg}\mu\Delta B}$ ← R

relative magnetic moment.

3A $|k\rangle = \frac{1}{\sqrt{N}}\sum_j e^{ijk} a_j^\dagger |0\rangle$

$$E_k = \langle k|H|k\rangle = -\frac{t}{N}\langle 0|\sum_m e^{-imk} a_m \sum_j (a_{j+1}^\dagger a_j + a_j^\dagger a_{j+1}) \sum_l e^{ilk} a_l |0\rangle$$

$$= -\frac{t}{N}\langle 0|\sum_m e^{-imk} a_m \sum_j \left(a_{j+1}^\dagger e^{ijk} + a_j^\dagger e^{i(j+1)k}\right)|0\rangle$$

$$= -\frac{t}{N}\langle 0|\sum_j e^{ijk} e^{-i(j+1)k} + e^{-ijk} e^{i(j+1)k}|0\rangle$$

$$= -\frac{t}{N}\sum_j \langle 0|e^{-ik} + e^{ik}|0\rangle = -2t\cos(k)$$

3B. $E_k \simeq -2t\left(1 - \frac{k^2}{2}\right) = -2t + tk^2 \Rightarrow \frac{\hbar^2}{2m} = t \Rightarrow m = \frac{\hbar^2}{2t}$

3C. k is the phase shift across one site $\Rightarrow e^{iKa} = e^{ik} \Rightarrow$

$Ka = k$, we have $E_k = tK^2a^2 = \hbar^2 K^2/2m$

$\Rightarrow m = \hbar^2/2ta^2$. this time the dimension is correct.

4A. $U = 0$ $|g_1\rangle = \left[\frac{1}{\sqrt{3}}(a_1^\dagger + a_2^\dagger + a_3^\dagger)\right]^3|0\rangle$

$t = 0$ $|g_2\rangle = a_1^\dagger a_2^\dagger a_3^\dagger |0\rangle$

$\frac{1}{3} * \frac{1}{3} \times \frac{1}{3} \times 6 + 3 * \frac{1}{3} * \frac{1}{3} * \frac{2}{3} * 2 = \frac{2}{3}$

$$\langle g_1|H|g_1\rangle = -6t + \frac{U}{2} 3 \langle g_1|n_1(n_1 - 1)|g_1\rangle = -6t + U$$

$\langle g_2 | H | g_2 \rangle = 0 + 0$

$\Rightarrow$ transition happens at $U = 6t$.

4B. $\bullet \,|\, \bullet \,|\, \bullet$ Ways to arrange 3 particles and 2 walls $C_2^5 = 10$.

4C. $S|300\rangle$, $S|210\rangle$. $S|111\rangle$. 3 dimensions