Lecture 12. Analog meet digital & Gates p226 electronics

Types of digital standards:
TTL (transistor transistor logic):
74 series, standard up to 100MHz, 5V on, 0V off, (1Ghz available)
CMOS (complementary metal oxide semiconductor):
low power, 100MHz, noise immune, 5/10/15V on, 0V off
ECL (emitter coupled logic): 500MHz, -0.9V on, -1.75V off

Remake:
1. TTL sinks current of a fraction of 1mA at low position. Fanout of 10 or less!
2. CMOS needs no current << μA
3. Edge sensitive logic should not be driven with slow signals.
   Switch bounce: Schmitt trigger, bistable positive feedback. See below.
4. Entry into digital circuits: comparator.
5. Is op amp the same as a comparator? Not quite.

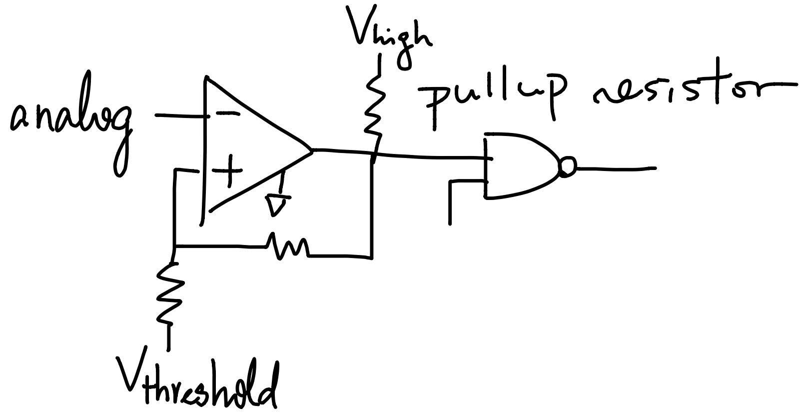
Difference: speed, noise, saturation...

Can one use an op amp as a comparator? You can, but need to know...

Diff optimization: with and without saturation

Comparator: important to have a pull up resistor!! Because it typically comes with an open collector and grounded emitter output.

Comparator

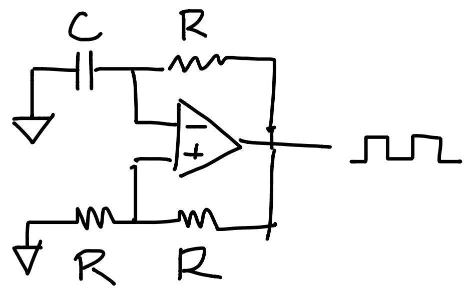


Why does it need pull-up resistor?

Comparators typically have an NPN output transistor with open collector and ground emitter. Below is one example.

Why are they designed this way??

Relaxation oscillator



Combinational logic and Boolean algebra

Any veriable $A = 0$ or 1.

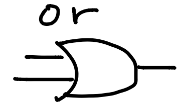
Operators:

truth table

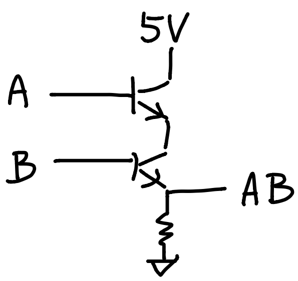
and

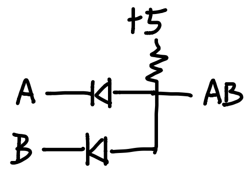
AB	$A=0$	1
$B=0$	0	0
1	0	1

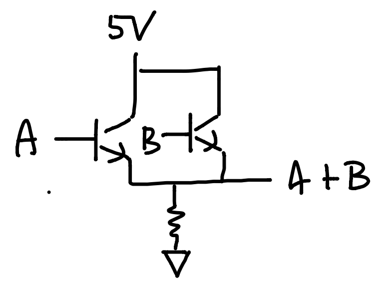
or

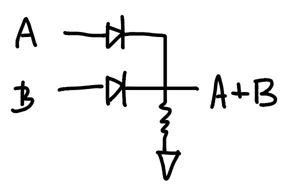


$A+B$	$A=0$	1
$B=0$	0	1
1	1	1

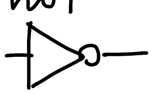




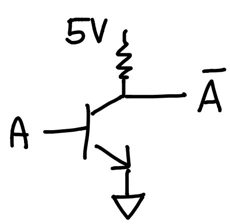




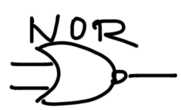
not



	$A=0$	1
$\bar{A}$	1	0

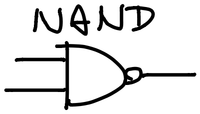


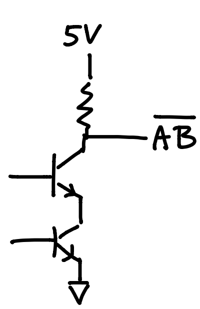
NOR



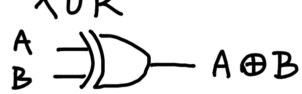
$\overline{A+B}$	0	1
0	1	0
1	0	0

NAND





XOR



	$A=0$	1
$B=0$	0	1
1	1	0

Boolean Algebra

A and $B \equiv AB$ Similar to a product

A or $B \equiv A+B$ Similar to addition

Not $A \equiv \bar{A}$, $\bar{\bar{A}} = A$

$\overline{AB} = \bar{A} + \bar{B}$ NAND

$\overline{A+B} = \bar{A}\,\bar{B}$ NOR

inverse element: $\triangle$
identity element: ✓
Associativity: ✓
commutativity: ✓
distributivity: ✓

Distributive law $A(B+C) = AB + AC$

$AA = A$ $\bar{A}A = 0$ $A+1=1$ $A1 = A$ $A+0=A$ $A0=0$.

exercise: $\overline{\overline{A+BC} + A\bar{B}}$

$= \overline{\overline{A+BC}}\;\overline{\overline{A\bar{B}}} = (A+BC)(A\bar{B})$

$= AA\bar{B} + \cancelto{0}{BCA\bar{B}} = A\bar{B} + 0 = A\bar{B}$

How many different gates do you need to do everything?

How would construct an arbitrary truth table

B \ A	0	1
0	X	Y
1	Z	W

$2^{2^2} = 16$ possible truth tables

If one can apply not-gate to A, B, F, we only have

X	X
X	X

$X = 1$ or 0, 2 config. — trivial

X	X
X	Y

$X \neq Y$, 8 config. — AND/OR

X	Y
Y	X

$X \neq Y$, 2 config. — XOR

X	Y
X	Y

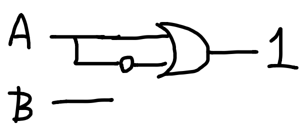
$X \neq Y$, / — trivial

So it looks like 4 gates are needed: 

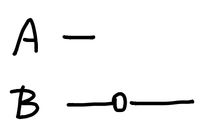
Actually, $AB = \overline{\bar{A} + \bar{B}}$, $A + B = \overline{\bar{A}\,\bar{B}}$, so we only need 3?

Example:

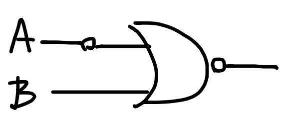
B \ A	0	1
0	1	1
1	1	1



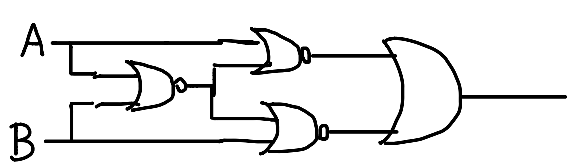
	0	1
0	1	1
1	0	0



	0	1
0	0	1
1	0	0



	0	1
0	0	1
1	1	0



$\Rightarrow A \oplus B = \overline{(A + \overline{(A+B)})} + \overline{(B + \overline{(A+B)})}$

So we only need 2 gates 

Questions: (i) It turns out NOT gate can be constructed by NOR, so there is only one universal gate.

$$\overline{(A+A)} = \bar{A}.$$

(ii) Can you construct a general gate when XYZW are given as variables?

(iii) How many gates are needed to accomplish a task of length $N \gg 1$?

Find an element. $\sim \Theta(1)$

Binary search $\sim \Theta(\log N)$

$X > y$? $\sim \Theta(N)$

Bubble sort $\sim \Theta(N^2)$

Matrix multiplication $\sim \Theta(N^3)$

Fracterization $\sim \Theta(2^{N^{1/3}}) \rightarrow N^3$ with QC.

Traveling salesman $\sim \Theta(N^N)$

Connection to Quantum computation

Universal quantum computation can be decomposed into single spin rotation and control-not gate.

Y \ X	0	1
0	0	1
1	1	0