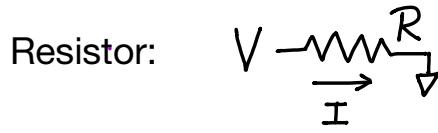
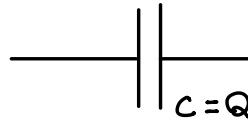


4/4/2019. Capacitors and inductors

Resistor:   $V = IR$

Capacitor: what is capacitance?

 how many charges  $Q$  one needed to raise the voltage  $V$  by 1 volt.

$Q$ : charge,  $V$ : voltage,  $I$ : current,  $C$ : capacitance

$$Z : \text{impedance} = \frac{V}{I}$$

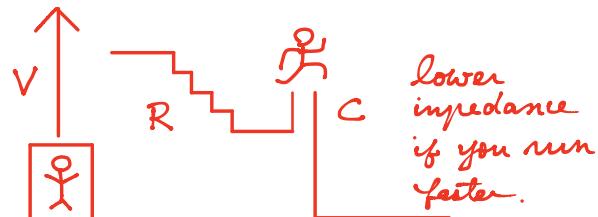
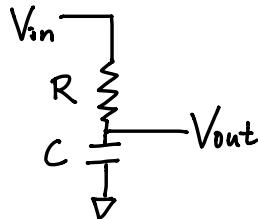
$$\text{let } V = V_0 e^{i\omega t}$$

$$Q = CV = CV_0 e^{i\omega t}$$

$$I = \frac{dQ}{dt} = CV_0 i\omega e^{i\omega t}$$

$$\Rightarrow Z_C = \frac{V_0 e^{i\omega t}}{CV_0 i\omega e^{i\omega t}} \Rightarrow Z_C = \frac{1}{i\omega C}$$

RC circuit: low-pass filter



Frequency domain calculation

$$V_{in} = IR + IZ_C = I(R+C)$$

$$V_{out} = V_{in} \frac{Z_C}{R+Z_C} = V_{in} \frac{1}{1+i\omega C}$$

$$\text{AC gain: } G_{AC} = \frac{V_{out}}{V_{in}} = \frac{1}{1+i\omega C}$$

$$G(\omega) \equiv |G_{AC}| = \frac{1}{\sqrt{1+\omega^2/\omega_0^2}}$$

$$\text{Bandwidth } f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi RC}$$

Time domain calculation

$$V_{in} = IR + Q/C$$

$$\dot{V}_{in} = iR + I/C$$

$$\Rightarrow V_o = Q/C = V_{in} (1 - e^{-t/RC})$$

$$\text{DC gain: } \frac{V_{out}}{V_{in}} = 1 - e^{-t/RC} \rightarrow 1$$

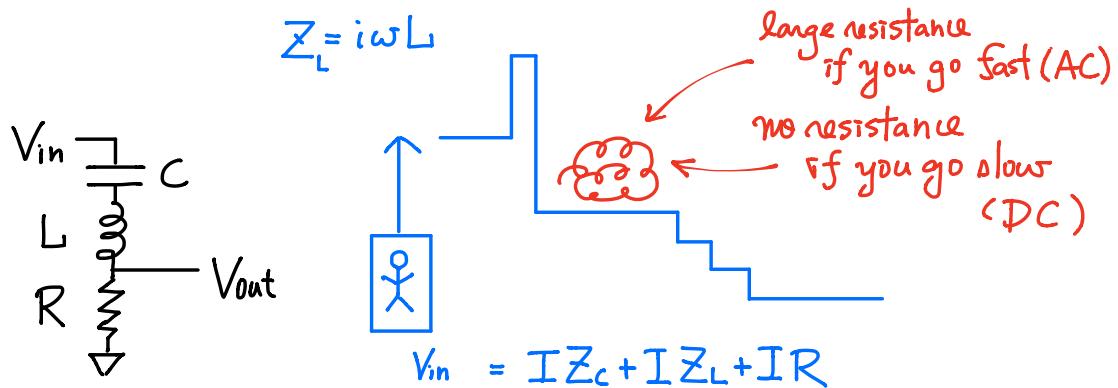
Summary : Capacitor is low pass.

DC goes       $G(\omega \rightarrow 0) = 1$       low freq.      ✓

$G(\omega = \omega_0) = \frac{1}{\sqrt{2}}$       mid-freq      △

AC    X       $G(\omega \gg \omega_0) = \frac{\omega_0}{\omega}$       high freq.      X

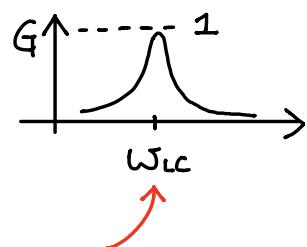
Inductor ————— DC go , AC no-go.



$V_{out} = IR$       x: reactance

$V_{out} = V_{in} \frac{R}{R + iX}$        $X = \omega L - \frac{1}{\omega C}$

$$\text{Gain } G(\omega) = \left| \frac{V_{out}}{V_{in}} \right| = \frac{\frac{\omega^2}{\omega_0^2}}{\sqrt{\left[ \left( \frac{\omega}{\omega_{LC}} \right)^2 - 1 \right]^2 + \frac{\omega^2}{\omega_0^2}}}$$



Resonance occurs when  $X=0 \Rightarrow \omega L = \frac{1}{\omega C}$   
 $Z_L$  cancels  $Z_C$        $\Rightarrow \omega_{LC} = (LC)^{-1/2}$