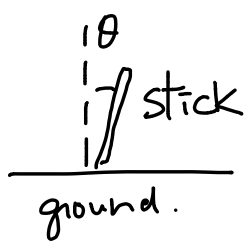
Lecture 9. Feedback 1. 4.30.2019

This is perhaps the most important concept of electronics.

Feedback is essential for reaching a well-defined goal.

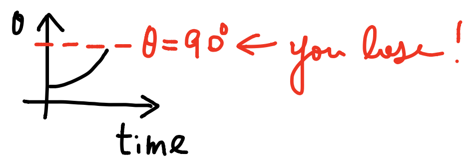
Example: balance a stick

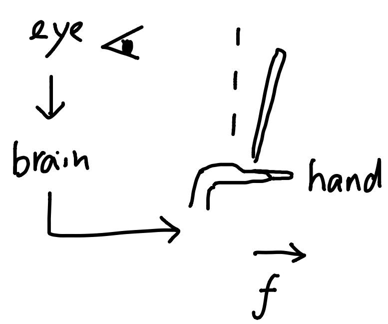


goal: minimize θ

if $\theta \neq 0$, Newton says

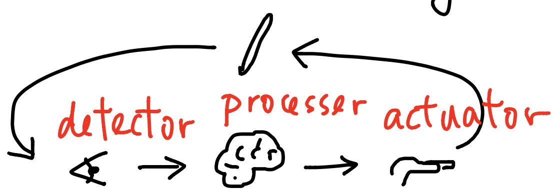
$I\ddot{\theta} = mg\sin\theta \Rightarrow$ unstable





$\Rightarrow$ One strategy: move the bottom point so you can reduce θ before it reaches 90°.

Back to stick balancing



Abstraction

noise (say, a mosquito nearby)

$$\begin{tikzcd} & \text{system} \arrow[dl, bend right] & \\ \text{detector} \arrow[r] & \text{processor} \arrow[r] & \text{actuator} \arrow[ul, bend right] \end{tikzcd}$$

Quantification

Sys. Detect. process. action

$$\begin{tikzcd} n \arrow[r] & \boxed{S} \arrow[r, "\theta"] & \boxed{D} \arrow[r] & \boxed{P} \arrow[r] & \boxed{A} \arrow[llll, bend left] \end{tikzcd}$$

Negative feedback

simplest feedback

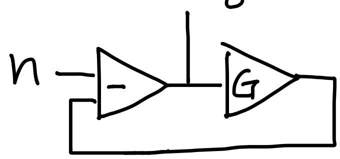
$\boxed{S}$ can be simply $\xrightarrow{n} \boxed{-} \rightarrow \theta = n - A$, $\uparrow A$

Ideal detector $\boxed{D} = \boxed{1}$

Ideal actuator $\boxed{A} = \boxed{1}$

Even though ideal $\boxed{D}$ & $\boxed{A}$ never exist, we still define $\boxed{D} - \boxed{P} - \boxed{A} \equiv \boxed{G}$. $G = P$ if $D = A = 1$.

error signal θ : deviation from our target (setpoint).



$$\theta = n - G\theta \Rightarrow \theta = \frac{n}{1+G}$$

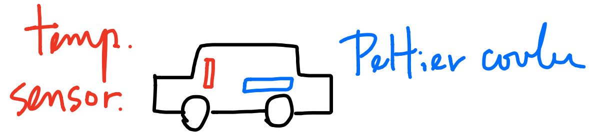
Without feedback ($G=0$), we have $\theta = n$. So noise is suppressed by $(1+G)$. Thus to suppress the noise we just make G infinite. Does that work?

No, it will be unstable, same reason you fell when you learned biking for the 1st time.

Solution to stablize an upside-down stick? Or temperature in a car?

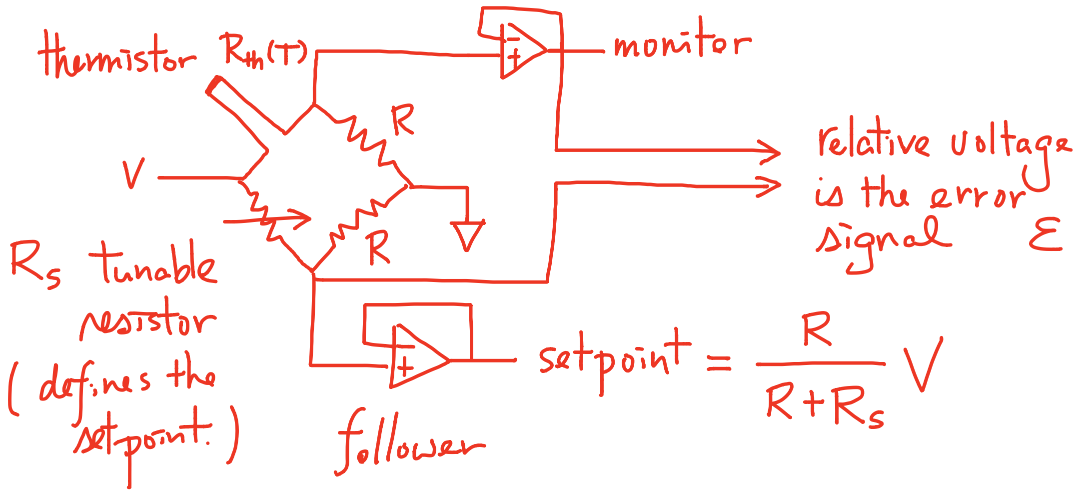
A concrete example: Temp servo you will work on:

Air conditioning:



temperature sensing:

Wheatstone bridge



$$\varepsilon = V \frac{R}{R+R_{th}} - V \frac{R}{R+R_s}$$

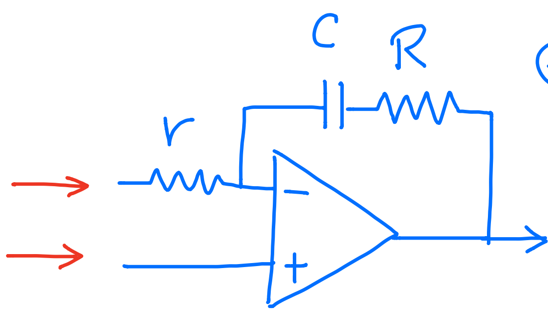
if $R_{th} = R_s + \Delta R$, $\Delta R \ll R_s$ $\Rightarrow$ $\varepsilon = V \frac{R}{R+R_s} \frac{\Delta R}{R+R_s}$

Error signal $\varepsilon = 0$ when $R_{th}(T_0) = R_s$ ← temp setpoint.

$$\Rightarrow \varepsilon = \frac{V}{R+R_s} \frac{1}{R+R_s} R_{th}'(T_0)(T-T_0).$$

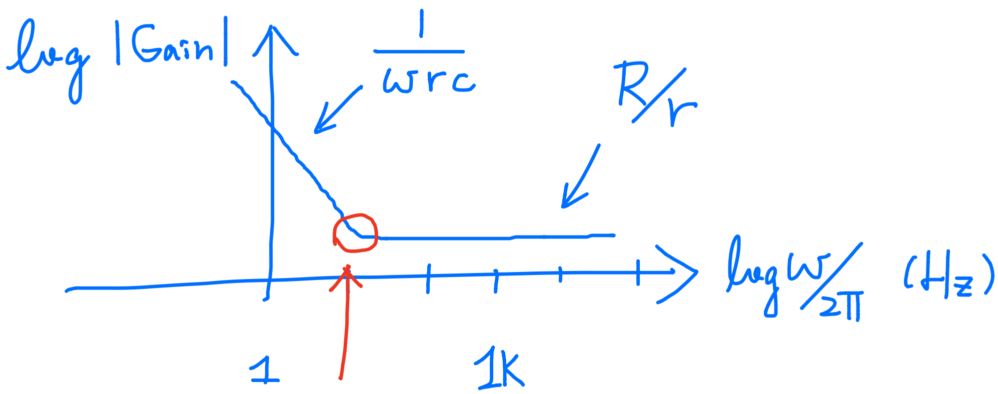
※ Error signal $= 0$ when $T = T_0$.

2nd part: Processing



$$G = -\frac{Z_2}{Z_1} = -\frac{R + \frac{1}{iwC}}{r}$$

$$V_{out} = G\, V_{in}$$

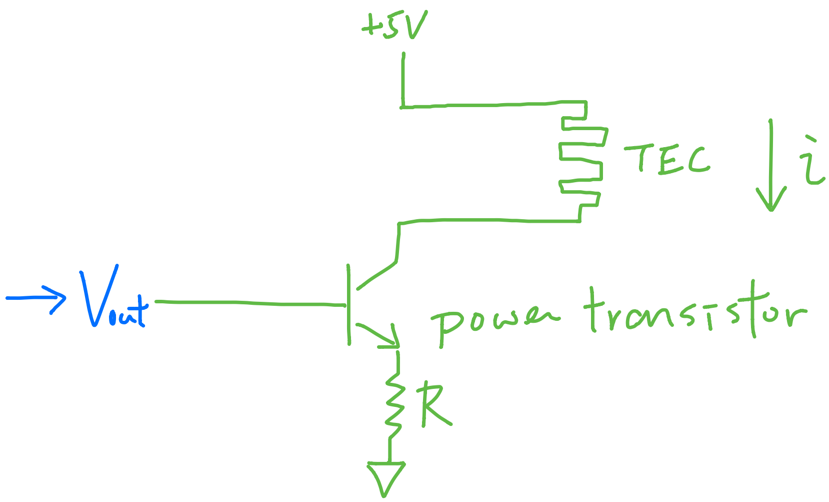


$$\frac{1}{wrC} = \frac{R}{r} \Rightarrow w/2\pi = 1/2\pi RC$$

As large gain as possible @ DC.

Lower gain @ higher freq since no point to react too fast. System cannot follow.

3rd part: Actuation



$$i = \beta i_b = \beta i_0 \left(e^{(V - i_b R)/kT} - 1\right)$$

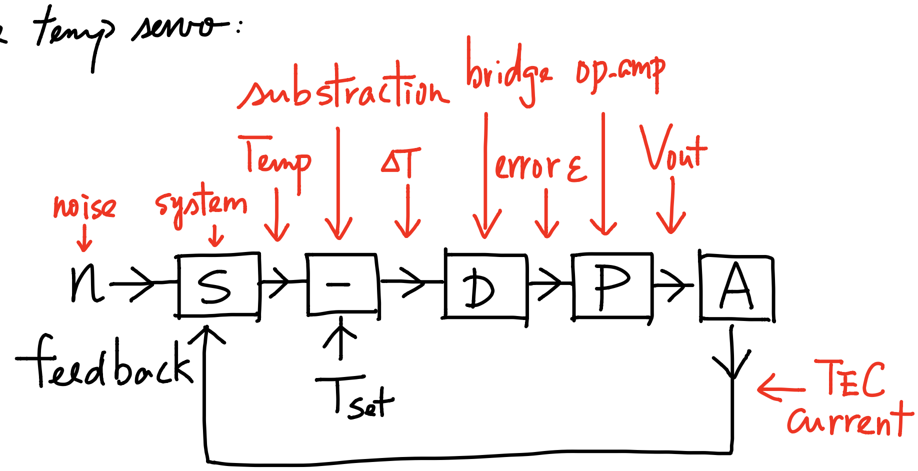
$$\delta i = i\, e^{V/kT}\, \delta V$$

Finally we need to check the direction of feedback, make sure we have negative feedback.

$$T \nearrow \Rightarrow R_{th} \searrow \Rightarrow V_{out} \nearrow \Rightarrow i \nearrow \Rightarrow T \searrow$$

Al temp   thermistor   feedback   actuator   Al temp.

Analyze the temp servo:



$\boxed{-}$: substraction : $\Delta T = T - T_{set}$

$\boxed{D}$: $\varepsilon = G_D \Delta T$, G_D = constant.

$\boxed{P}$: $V = G_p \varepsilon$, $G_p = \frac{Z}{Z_1} = \frac{R + 1/i\omega C}{r}$

$\boxed{A}$: $i = G_A V$

$\boxed{S} = T(n, i)$ is the physics of the Al block.

Overall we have, for small deviations from the set point.

$$T = T_{set} + n - G_s \Delta i \Rightarrow \Delta T = n - G_s \Delta i$$

$$\Rightarrow \varepsilon = G_D \Delta T. \quad \Delta i = G_A \Delta V = G_A G_p \varepsilon$$

$$\Rightarrow \Delta T = n - G_s G_A G_p G_D \Delta T$$

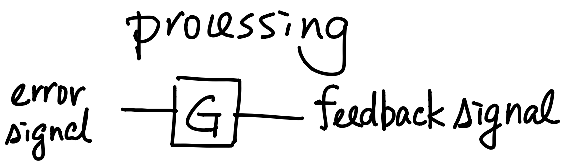
$$\Rightarrow \Delta T = \frac{n}{1 + G_s G_A G_p G_D}.$$

Feedback theory:
how much can we reduce n?

Feedback theory

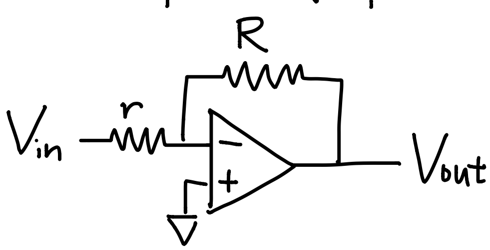
Given that we like to have large gain to suppress the noise, and limited performance on detection, actuation and knowledge of the system, how do we design a feedback to suppress the noise as much as possible?

A generic solution: PID controller.

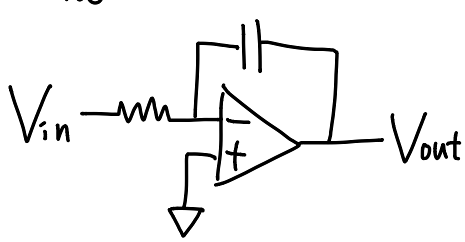


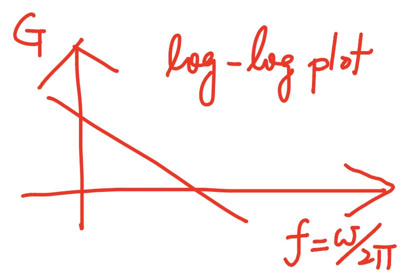
$V_{out} = G V_{in}$

$G = G_p$ proportional gain

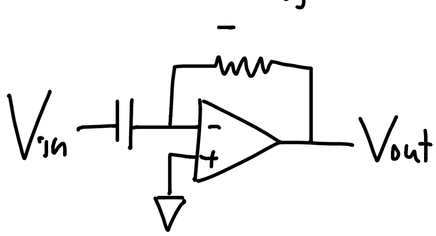


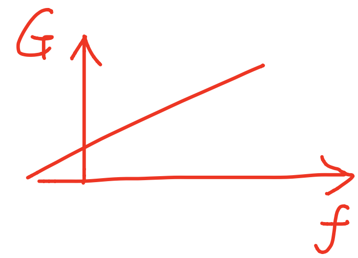
or $G = \frac{G_I}{i\omega}$ integration gain



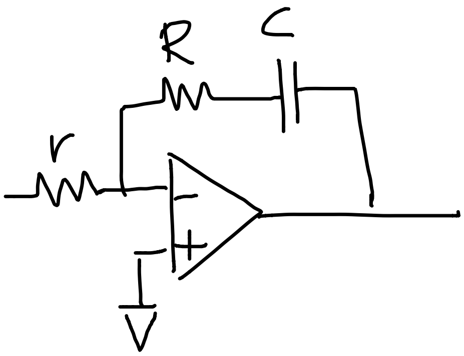


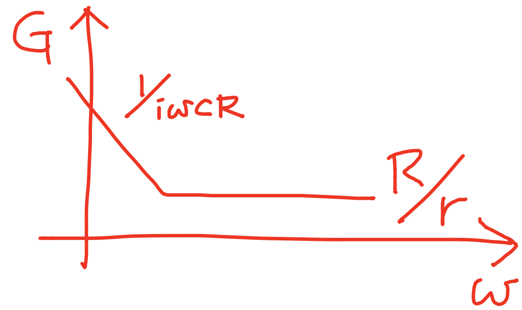
or $G = i\omega G_D$ differential gain



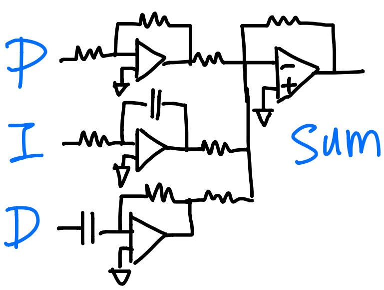


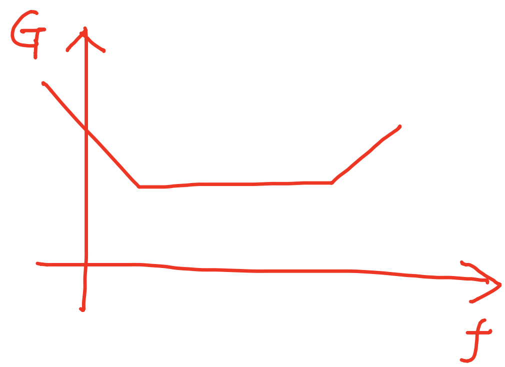
or $G = G_p + \frac{G_I}{i\omega}$ P-I control (Labsession)





or everything P-I-D control





Example:

Proportional gain

Use present value

Error $\varepsilon = n - G\varepsilon$

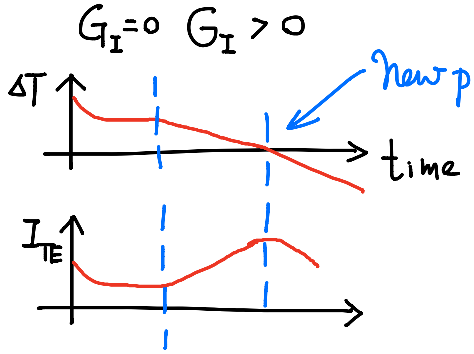
$$= \frac{n}{1+G}$$

Integration gain: $\varepsilon = n - G_I \int \varepsilon(t)\,dt$

$$\varepsilon_\omega = n_\omega - \frac{G_I}{i\omega} n_\omega$$

$$= \frac{n_\omega}{1 + G_I/i\omega}$$

Sum over the past.



New problem: system is unaware that we should slow down approaching the setpoint.

Differential gain: $\varepsilon = n - G_D\, d\varepsilon/dt$

predicting the future