Physics 143b: Honors Waves, Optics, and Thermo

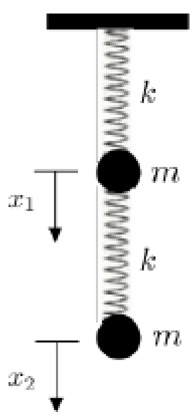
Spring Quarter 2020
Problem Set #2
Due: 11:59 pm, Thursday, April 23. Please submit to Canvas.

1. **MATH** (10 points each) What will happen if you couple an underdamped oscillator to an overdamped oscillator? In HW1 problem 1 we solved $x'' + 3x' + 2x = 0$ and $x'' + 2x' + 2x = 0$. One describes an overdamped oscillator and the other one is underdamped. Now we couple them as

$$x'' + 3x' + 2x = 2y$$
$$y'' + 2y' + 2y = 2x$$

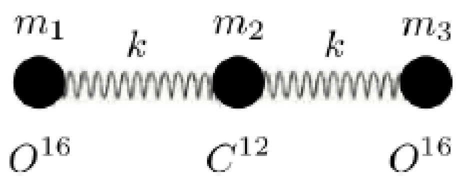
   (a) Write the equations in the vector-matrix form as $\vec{x}''(t) + \hat{\gamma}\vec{x}'(t) + \widehat{M}\vec{x}(t) = 0$, where $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ and determine the matrices $\hat{\gamma}$ *and* $\widehat{M}$.
   (b) Determine the four eigen-frequencies and is the associated motion under-, critical- or overdamped?
   (Hint: You may find the roots of the determinant numerically or online.)
   (Hint: You will also find a strange normal mode, explain the corresponding solution.)
   (c) Given two identical critically damped oscillators and couple them. Describe whether the normal modes are under-, critical- or over-damped?
   (Hint: You may introduce εx and εy to the equations to couple the oscillators.)

2. **Two masses on vertical springs** (10 points each) Two identical masses are attached to two massless springs as shown. Considering only motion in the vertical direction, solve for the motion of each mass about their equilibrium positions.



(A) Given the gravitational pull of *mg* and assuming no friction, write down the differential equations that describe the motion of the masses in terms of the deviations of their positions x_1 and x_2 away from the equilibrium position in the direction shown in the figure.
Show that gravitational pull does not show up in the equations.
(B) Determine and describe the eigenmodes and their frequencies.
(C) At $t=0$, the lower mass receives a kick with velocity $x_2'(0)=v_2$, while $x_1(0)= x_1'(0)= x_2(0)$, determine their subsequent motion $x_1(t)$ and $x_2(t)$.

3. **Vibrations of a CO2 molecule (10 points each)** The CO2 molecule can vibrate like a system made up of a central mass m2 connected by equal springs of spring constant k to two identical masses m1=m3=m.



(A) First we consider linear motion of the 3 atoms along the symmetry axis. Write down the differential equations that describe their motion x_1, x_2 and x_3 and determine the eigenfrequencies.
(B) Describe the eigenmodes. In principle there should be 3 eigenmodes for 3 oscillators. Explain why there are only two vibrational modes in this case.

4. **Three-body chase (10 points each)** In the forest, Antandra, Brutus and Cecilia are chasing each other. Antandra accelerates toward Brutus, but away from Cecilia; Brutus accelerates toward Cecilia, but away from Antandra; Cecilia accelerates toward Antandra, but away from Brutus. Assume their accelerations/deccelerations are proportional to their separations; for instance, $a_A = x_A'' = \kappa\,(x_B - x_A) - \kappa(x_C - x_A)$ where x_A, x_B *and* x_C are the locations of Antandra, Brutus and Cecilia. They share the same κ.

(A) Assume their motions are one dimensional, write down their equation of motion in the vector-matrix form of $\vec{x}''(t) = \widehat{M}\vec{x}(t)$, where $\vec{x} = \begin{pmatrix} x_A \\ x_B \\ x_C \end{pmatrix}$, and determine the matrix.
(B) Describe the 3 eigenmodes of their motion qualitatively.