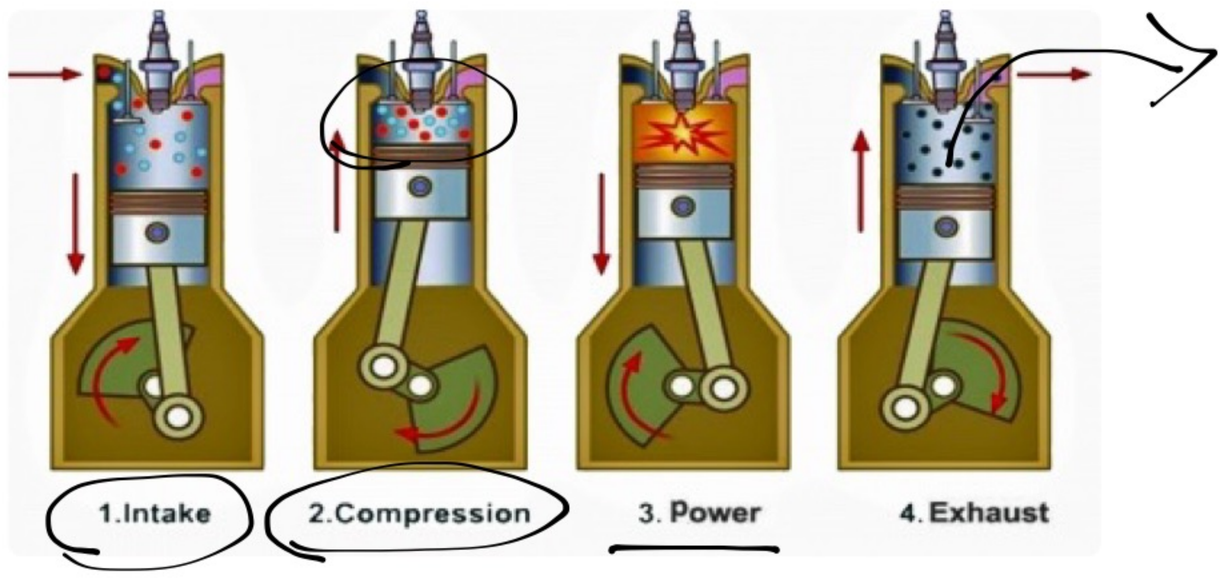
Lecture 15-1 Heat engine

1st see the video on how a modern ICE works:

Internal Combustion Engine (ICE)



Intake Stroke

Compression Stroke

Ignition Stroke

Power Stroke

Exhaust Stroke

In this lecture we will understand the heat engine based on simple models: ideal gas

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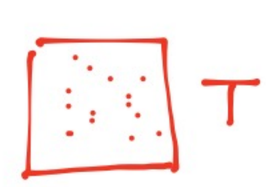
Background knowledge & approximation

Approximate air as ideal gas $\boxed{PV = NkT}$

Energy of the gas: $\boxed{U = N\gamma kT,}$ $\gamma = \frac{3}{2}$ for atom

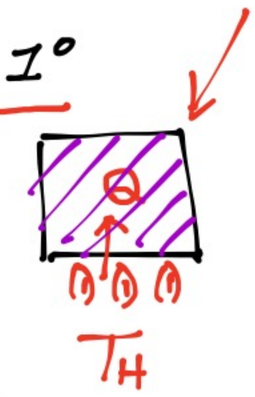
$\approx \frac{5}{2}$ for air

$\frac{1}{2}kT = \frac{1}{2}mv_x^2$



Isochoric process ($V=$const.)

Specific heat: heat needed to raise unit mass by 1°



$$\Rightarrow C_v = \frac{1}{M}\frac{dQ}{dT} = \frac{1}{M}\frac{dU}{dT} = \frac{N}{M}\gamma k = \frac{1}{m}\gamma k$$

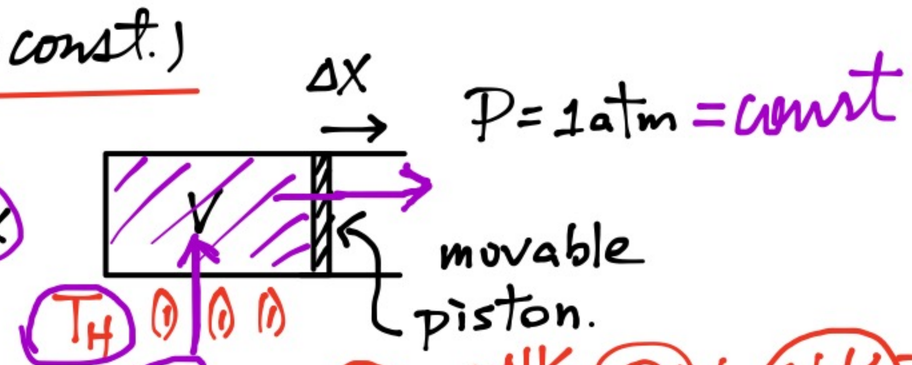
total mass M; mol. mass m

Under the assumption the volume does not change $\Delta V=0$.

$\Rightarrow C_v = \boxed{\gamma \frac{1}{m} k}$ is called isochoric specific heat. C_v

$V=$const

If pressure is held constant ($p=$const.)



$$\Delta U = \Delta Q - F\Delta X = \Delta Q - PA\Delta X$$
$$= \Delta Q - P\Delta V$$

We need more energy $C_p = \frac{1}{M}\frac{\Delta Q}{\Delta T} = \frac{1}{M}\left(\frac{\Delta U}{\Delta T} + P\frac{\Delta V}{\Delta T}\right)$ $\quad \frac{NK}{P}$ $\quad PV = NKT$

$$= \frac{N}{M}\gamma k + \frac{N}{M}k = \frac{N}{M}(\gamma+1)k = \frac{\gamma+1}{\gamma}C_v$$

$C_p : C_v = \gamma+1 : \gamma$

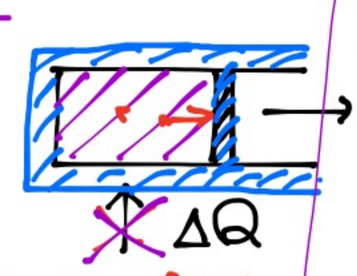
$\Rightarrow C_p = \frac{\gamma+1}{\gamma}C_v$ is isobaric specific heat.

$C_v = \frac{1}{M}\left(\partial_T Q\right)_V$ pressure pot

$C_p = \frac{1}{M}\left(\partial_T Q\right)_P$ regular pot

adiabatic process

The third process is for an isolated system ($\Delta Q = 0$)



$U \propto T$ $\quad U = \gamma NKT$

$$\Delta U = -P\Delta V = \gamma NK\Delta T = \gamma\Delta(PV)$$

$$\Rightarrow P\Delta V + \gamma P\Delta V + \gamma V\Delta P = 0$$

$\frac{1}{PV}$

$$\Rightarrow (\gamma+1)P\Delta V + \gamma V\Delta P = 0.$$

$$\Rightarrow (\gamma+1)\frac{\Delta V}{V} + \gamma\frac{\Delta P}{P} = 0 \Rightarrow PV^{\frac{\gamma+1}{\gamma}} = \text{const.} \Rightarrow PV^{C_p/C_v} = c$$

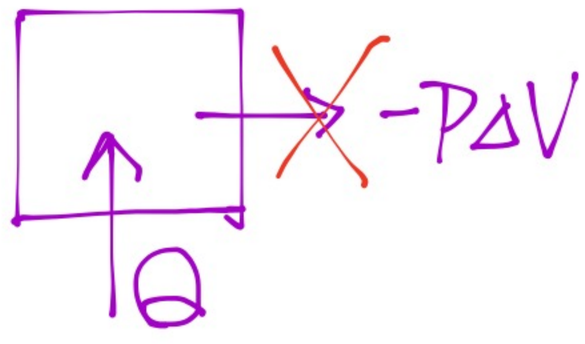
$$\Delta(P^\gamma V^{\gamma+1}) = \gamma P^\gamma V^{\gamma+1}\left(\frac{\Delta V}{V} + \frac{\Delta P}{P}\right) = 0$$

Lastly, if $T=$ const. $PV=$ const. $\Delta U = 0$. isothermal process

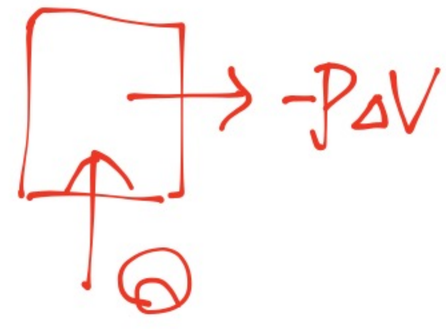
$$\Delta U = \Delta Q - P\Delta V = 0 \Rightarrow \Delta Q = P\Delta V$$

if you expand a gas, it absorbs heat from environment. $T=$

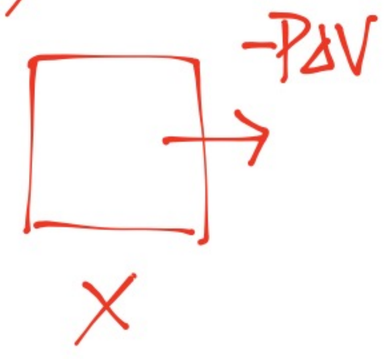
(1) $\Delta V = 0$



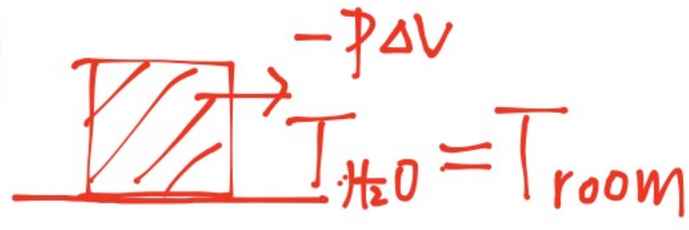
(2) $P = 0$



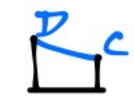
(3) adiabatic

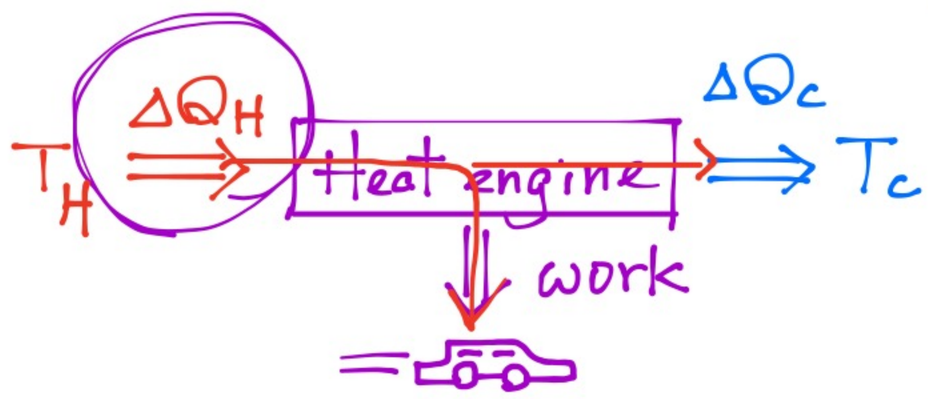


(4)

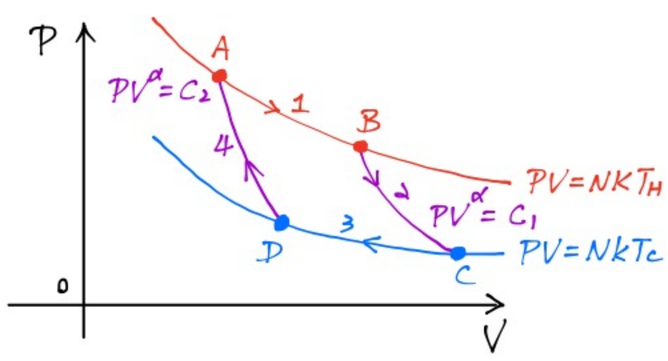


Heat absorbed from the hot source $\Delta Q_H = \int_A^B P\,dV =$ 

Heat released to the cold plate $\Delta Q_C = \int_C^D P\,dV =$ 



$$\text{Energy conversion efficiency} = \frac{\Delta\,\text{work}}{\Delta Q_H} = \frac{\text{ABCD area}}{\text{AB area}}$$



$$\text{AB area} = \int_A^B P\,dV = NKT_H \int \frac{1}{V}\,dV = \boxed{NKT_H \ln\frac{V_B}{V_A}}$$

$$\text{BC area} = \int_B^C P\,dV = C_1 \int_B^C V^{-\alpha}\,dV = C_1 \frac{1}{-\alpha+1}\left(V_C^{-\alpha+1} - V_B^{-\alpha+1}\right)$$

Use $C_1 = P_B V_B^\alpha = P_C V_C^\alpha$, $C_2 = P_A V_A^\alpha = P_D V_D^\alpha$,

$P_A V_A = P_B V_B = NKT_H$. $P_D V_D = P_C V_C = NKT_C$

Show that (1) $\boxed{\frac{V_C}{V_D}} = \boxed{\frac{V_B}{V_A}}$ (2) DC area $= NKT_C \ln \boxed{\frac{V_C}{V_D}}$ (3) BC area = AD area

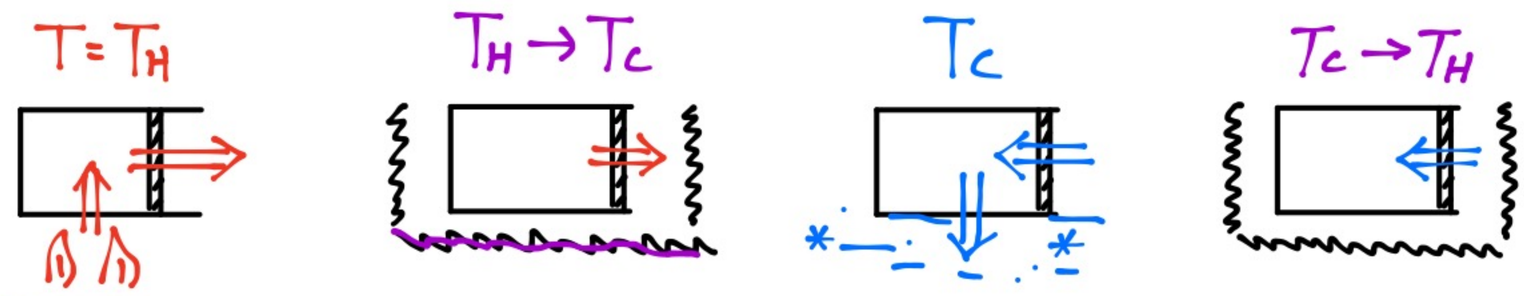
(4) ABCD area $= NK(T_H - T_C)\ln\frac{V_B}{V_A} \propto T_H - T_C$

(5) $\Delta Q_H =$ AB area $= NKT_H \ln\frac{V_B}{V_A} \propto T_H$ $\quad T_H \gg T_C \quad \varepsilon \to 1$

$\Rightarrow$ Carnot cycle efficiency $= \frac{\text{ABCD area}}{\text{AB area}} = \boxed{1 - \frac{T_C}{T_H}}$ is known to be the best efficiency for a thermal engine exchanging heat with 2 reseviors at T_H and T_C. Unless $T_C = 0$, we never reach 100%. $\quad 1 - \frac{300}{900}$

Lecture 15-2 Heat engines

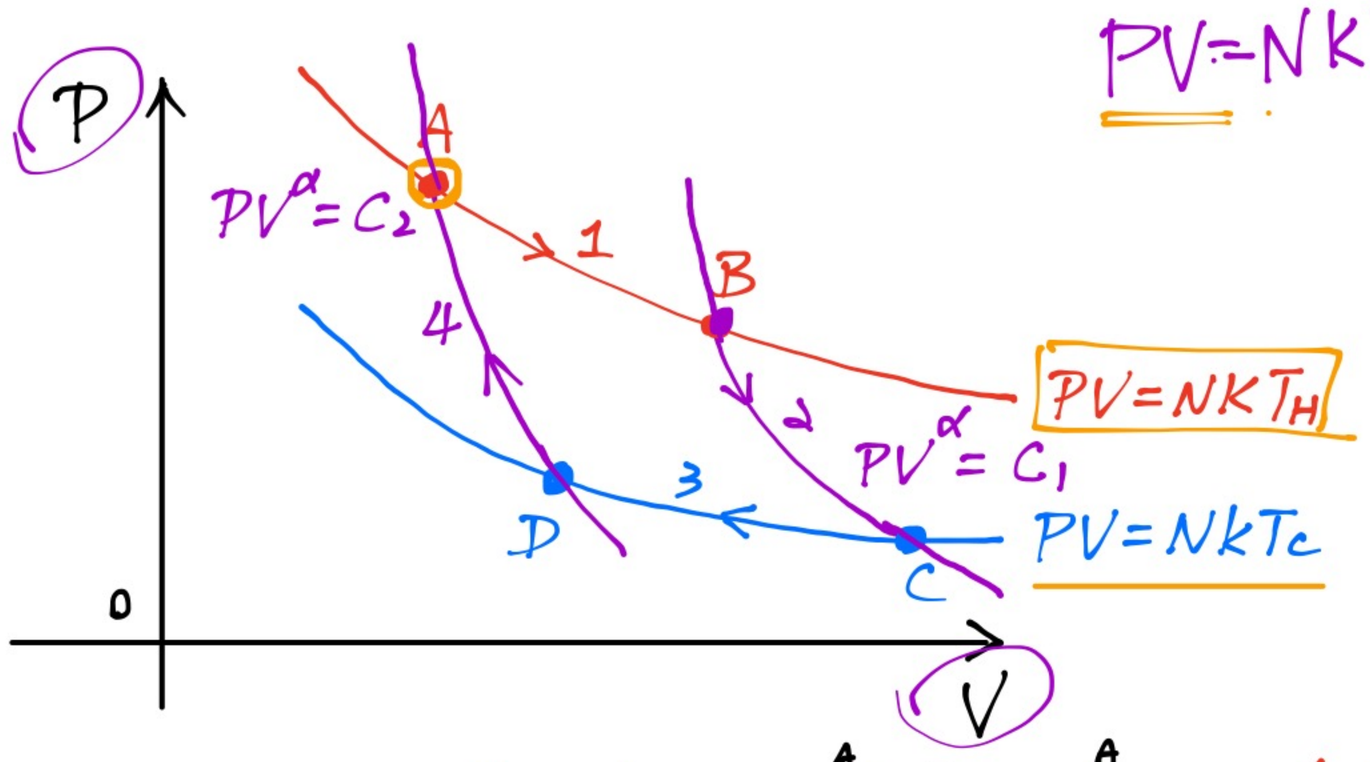
Simplest and illustrative example of motors driven by heat.

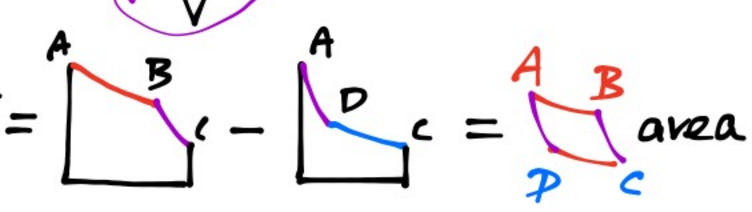


step 1: isothermal expansion	step 2: adiabatic expansion	isothermal compression	adiabatic compression
$\Delta Q > 0$ absorb heat	$\Delta Q = 0$	$\Delta Q < 0$ cool down	$\Delta Q = 0$
$P\Delta V > 0$ does work	$P\Delta V > 0$	$P\Delta V < 0$ negative work	$P\Delta V < 0$
$\Delta U = \Delta Q - P\Delta V = 0$ $\Rightarrow \Delta Q = P\Delta V > 0$	$\Delta U = -P\Delta V < 0$	$\Delta U = \Delta Q - P\Delta V = 0$ $\Delta Q = P\Delta V < 0$	$\Delta U = -P\Delta V > 0$
$PV = NkT_H = \text{const.}$	$PV^{\frac{\gamma+1}{\gamma}} = \text{const.}$ $\equiv PV^{\alpha} = C_1$	$PV = NkT_C = \text{const.}$ $\Delta U = 0$	$PV^{\frac{\gamma+1}{\gamma}} = \text{const.}$ $PV^{\alpha} \equiv C_2$

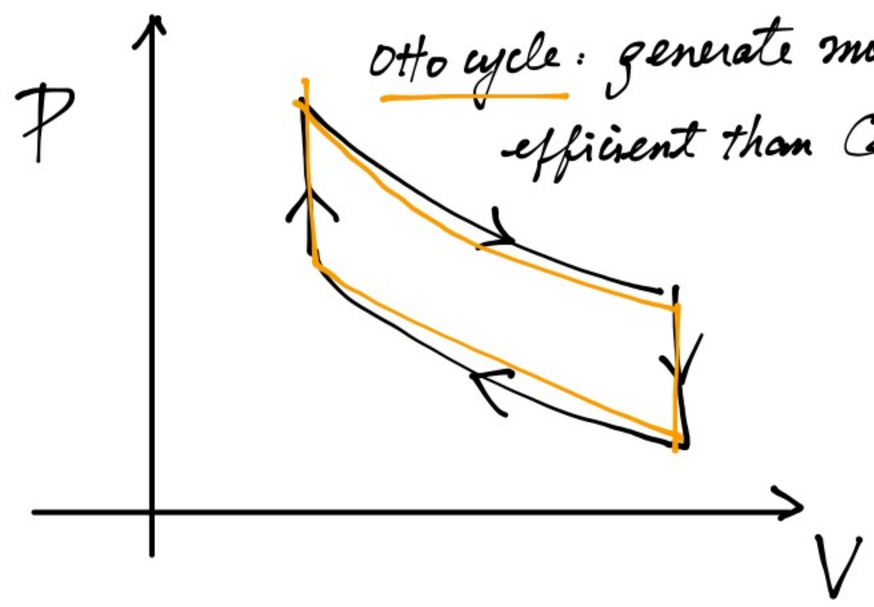
How much work does a Carnot engine do per cycle?

$PV = NKT =$



Work done by the system: $\int_A^B + \int_C^D PdV =$  area

Typical ICEs use $T_H = 600\,C \Rightarrow$ Carnot efficiency $= 55\%$. Our cars only reach 25% (Otto cycle), wasting 75% of the energy into heat.



Otto cycle: generate more energy, but even less efficient than Carnot cycle.