# Physics 143b: Honors Waves, Optics, and Thermo

Spring Quarter 2020

Practice Problem Set

1. Oscillations near equilibrium

   1.1 A mechanical metronome is a pendulum with a heavy mass M below the pivot point and a light mass m above the pivot point. The heavy mass is L away from the pivot point, and the light mass is l away.

   

   a. Determine the equation of motion of the metronome
   b. Determine the condition that it will not flip upside down.
   c. Show that you can tune its frequency by changing the location of the small mass.

   1.2 A particle is moving in the presence of a potential $f(x,y) = -f(\vec{k}\cdot\vec{x}) = -f_0\cos(x+2y+3z)$. At the origin, the potential reaches its minimum value.

   a. Determine the equation of motion of the particle by Taylor expanding the potential to the second order.
   b. Show that even though the curvature is positive along the x, y, z axes, $\partial_z^2 f > \partial_y^2 f > \partial_x^2 f > 0$, the particle is actually not confined when it is released near the origin.
   c. Show that the particle will escape along the direction that is perpendicular to $\vec{k}$.

   1.3 A mass m is attached to a spring with spring constant k. The system is hung vertically in equilibrium with an initial extension of $L_0 = \frac{mg}{k}$. At $t = 0$, we kick the mass gently so it acquires a velocity $\vec{v} = (v_x, v_y = 0, v_z)$.

   a. Determine the potential energy from both gravitational of the mass $V_g = -mgz$ and the spring $V_k = \frac{1}{2}k(x^2+z^2)$
   b. Determine the eigenmodes of the system
   c. Determine the dynamics of its length $L(t)$ and the swing angle $\theta(t)$ after the kick

2. Fourier expansion and transform

   2.1 $f(t) = e^{-a|t|}$, with $a > 0$. Show that $f(\omega) = \frac{2a}{a^2+\omega^2}$.

   2.2 $f(-T < t < T) = 1$ and $f(|t| > T) = 0$. Show that $f(\omega) = \frac{2\sin\omega T}{\omega}$.

   2.3 Let $f(x)$ be a function of period $2\pi$ such that $f(-\pi < x < 0) = 1$ and $f(0 < x < \pi) = 0$.

   a. Show that $f(x) = \frac{1}{2} - \frac{2}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)$
   b. Choose a proper value of $x$ and show that $\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\ldots\right)$.

3. Vector calculus

   3.1 $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$

   3.2 $\vec{\nabla}(\vec{A}\cdot\vec{B}) = \vec{A}\times(\vec{\nabla}\times\vec{B}) - (\vec{\nabla}\times\vec{A})\times\vec{B} + (\vec{B}\cdot\vec{\nabla})\vec{A} + (\vec{A}\cdot\vec{\nabla})\vec{B}$

   3.3 $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{A}\cdot(\vec{\nabla}\times\vec{B}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$

   3.4 $\vec{\nabla}\times(f\vec{A}) = f\vec{\nabla}\times\vec{A} + \vec{\nabla}f\times\vec{A}$

3.5 $\nabla^2(fg) = f\nabla^2 g + 2\vec{\nabla} f \times \vec{\nabla} g + g\nabla^2 f$
3.6 $\vec{\nabla} \cdot \left(\vec{\nabla} f \times \vec{\nabla} g\right) = 0$

4. 1D Wave equation $\partial_t^2 \psi = v^2 \partial_x^2 \psi$
Given the general solution $\psi = \sum A_n e^{ik_n x} e^{i\omega_n t}$, where $\omega_n = \pm v k_n$, what are the types of problems we may see in our surrounding?
4.a Rope jumping: $\psi(x_1, t) = f_1(t)$ and $\psi(x_2, t) = f_2(t)$, determine $\psi(x, t)$
4.b String instruments: $\psi(x, 0) = f(x)$ and $\psi\left(\pm\frac{L}{2}, t\right) = 0$, determine $\psi(x, t)$ and $\omega_n$.
4.c Wind instruments with 2 open ends: $\psi(0, t) = f(t)$ and $\psi_x\left(\pm\frac{L}{2}, t\right) = 0$ . Show that the eigen frequencies are the same as 4.b
4.d Wind instruments with 1 open and 1 close end: $\psi(0, t) = f(t)$ and $\psi(0, t) = \psi_x(L, t) = 0$ . Determine the eigen frequencies and show that there are even harmonics.