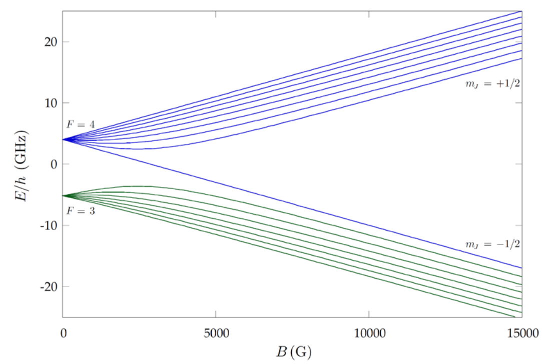
Physics 238: Atomic Physics
Fall Quarter 2021
Problem Set #3

Due: 12:20 pm, Thursday, October 28. Please submit in class.

1. **From Zeeman to Paschen-Back regime**

   Here we continue our model in Problem 1 in Homework #2 to understand atomic structure. In the presence of magnetic field, the ground states of an alkali atom with one valence electron *s*=1/2 and nuclear spin *i* split into as many as $(2i+1)^2$ magnetic levels. In Homework #2, we show that they can be labelled as $|F, m_F; s = \frac{1}{2}, i >$ when the magnetic field *B* is introduced perturbatively to the Hamiltonian $H = A\,\boldsymbol{s}\cdot\boldsymbol{i} - \boldsymbol{\mu}\cdot\boldsymbol{B}$, which suggests that F and m_F are good quantum numbers. Here in the Zeeman regime.



   (a) Paschen-Back regime occurs in the limit when Zeeman shifts is much greater than the hyperfine interactions $A\,\boldsymbol{s}\cdot\boldsymbol{i}$. Treating the hyperfine interactions as a perturbation, show that the eigenstates and eigenenergies are

$$|m_s, m_i; s = \frac{1}{2}, i >$$

$$E_{m_s m_i} = \frac{g}{2} B m_s + A m_s m_i$$

   (b) In the intermediate regime, determine the eigen-energies and compare to the attached figure for the case of cesium with $i = 7/2$.

2. **Doppler cooling**

Here we will consider how laser changes the motion of an atom. In the class, we showed how photon scattering can modify the motion of an atom. In particular, after scattering a photon, the averaged atomic kinetic energy changes by $\Delta K = \frac{1}{m}(p \cdot \hbar k + \hbar^2 k^2)$, where p and $\hbar k$ are the atomic and photonic momentum, respectively. We also showed from the density matrix calculation that the averaged photon scattering rate is given by $s = \Gamma \rho_{22}$, where $\rho_{22} \approx \frac{1}{2}\frac{I/I_s}{1+4\Delta^2/\Gamma^2}$ is the probability in the excited state in the presence of weak laser intensity $I \ll I_s$, Γ is the excited state decay rate. Here the laser detuneing $\Delta = \omega - \omega_0(p)$ is referenced to the resonance frequency of an atom with momentum p. We derived in the class $\omega_0(p) = \omega_0 + \frac{1}{m}\left(p \cdot \hbar k + \frac{1}{2}\hbar^2 k^2\right)$ and ω_0 is the resonance frequency of a pinned atom and the two terms in the paranthesis are the Doppler shift and recoil energy.

In the following we will consider one dimensional motion of an atom interacting with laser light:

(a) Determine the condition that the radiative force on an atom can be modeled as a damping $f = -\beta v$. Determine the detuning that yields the largest damping coefficient β.

(b) Consider two laser beams with the same frequency ω illuminate the atoms from $\pm x$ direction. Show that the atom can be *Doppler cooled* and derive the steady state equilibrium kinetic energy. Clarify the approximations you employ to obtain the result.

(c) Determine the fundamental limit of the Doppler cooling (the lowest kinetic energy one can reach by Dopper cooling).

3. Jayne-Cummings model (Dressed atom picture)

In the class, we discussed *|g, n+1>* can couple to *|e, n>*, where *n=0,1,2…* is the number of photons in the cavity and *g/e* refers to atom in the ground/excited state. The dressed atom Hamiltonian under RWA is described by the Jayne-Cummings model:

$$H = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega b^+ b + g^*(\sigma^+ b + \sigma^- b^+),$$

where $\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $\sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ and g^* is the coupling constant.

A. First we check whether this Hamiltonian is consistent with the semi-classical Hamiltonian $H = -\frac{\Delta}{2}\sigma_z + \frac{\Omega}{2}\sigma_x$ when the photon number is large $< b^+ b > \equiv n >> 1$. Write down the dressed atom Hamiltonian in the basis of $|e, n>$ and $|g, n>$:

   (Hint: the lower right corner of the Hamiltonian can be written as

$$H = \begin{pmatrix} < e,1|H|e,1> & < e,1|H|e,0> & < g,1|H|g,0> \\ < e,0|H|e,1> & < e,0|H|e,0> & < e,0|H|g,0> \\ < g,0|H|e,1> & < g,0|H|e,0> & < g,0|H|g,0> \end{pmatrix}$$

B. Calculate the eigenenergies for the lowest 7 states. You may assume $\Delta>0$.
C. Now let's include spontaneous emission, which permits decays from $|e,n>$ to $|g,n>$ by emitting a photon outside the cavity. Draw wiggling lines to show how an atom can decay toward lower states. Determine the frequencies of all emitted photons if an atom is initially prepared in $|e,2>$?