Physics 238: Atomic Physics
Fall Quarter 2021
Problem Set #5
Due: 12:20 pm, Thursday, November 18. Please submit in class.

1. Bose condensate near a wall

A. Given a wall $V(r<0)=\infty$ and $V(r>0)=0$, calculate the wavefunction of a condensate with chemical potential μ by solving the Gross-Pitaevskii equation,

$$\mu\Psi(r)=(\frac{p^2}{2m}+V(r)+g\,|\,\Psi(r)\,|^2)\Psi(r)\,.$$

(Hint: Given the boundary condition $\psi(0)=0$, the solution can be written as $\Psi(r)=\sqrt{n}\tanh\frac{r}{\sqrt{2}\xi}$. where $\xi=\sqrt{\frac{\hbar^2}{2m\mu}}$ is called the healing length, and n can be linked to the chemical potential.)

B. We know that single particle ground state in a box is $\Psi(x)\sim\sin kx$. How would you modify the potential such that the ground state wavefunction of a single particle is $\Psi(x)\approx$const.? (Hint: Use the result in A.)

Reference: Y. Castin and R. Dum, PRL 77, 5315, (1996)

2. Scattering by a square well

One effective potential to model finite range potential is a 3D square well potential $V(r<r_0)=-\frac{\hbar^2q^2}{2\mu}\equiv -D$ and $V(r>r_0)=0$. Consider the incoming spherical wave e^{-ikr}/r and the outgoing wave is $-Se^{ikr}/r$, where $S=e^{2i\delta}$ is the scattering matrix and δ is the s-wave scattering phase shift.

A. show that the scattering phase shift is $\delta=-kr_0+\tan^{-1}\frac{k\tan\sqrt{q^2+k^2}r_0}{\sqrt{q^2+k^2}}$.

B. Determine scattering length a in the low scattering energy limit $k\to 0$.

(Hint: Determine the location where wavefunction vanishes in the low energy limit.)

C. Plot the scattering phase shift and scattering length vs. the depth D.

~~D. By taking the limit $r_0\to 0$, we may introduce the pseudo potential $V(r)=-g\,\delta(r)$ to model low energy scattering with scattering length a. Determine the value of g.~~