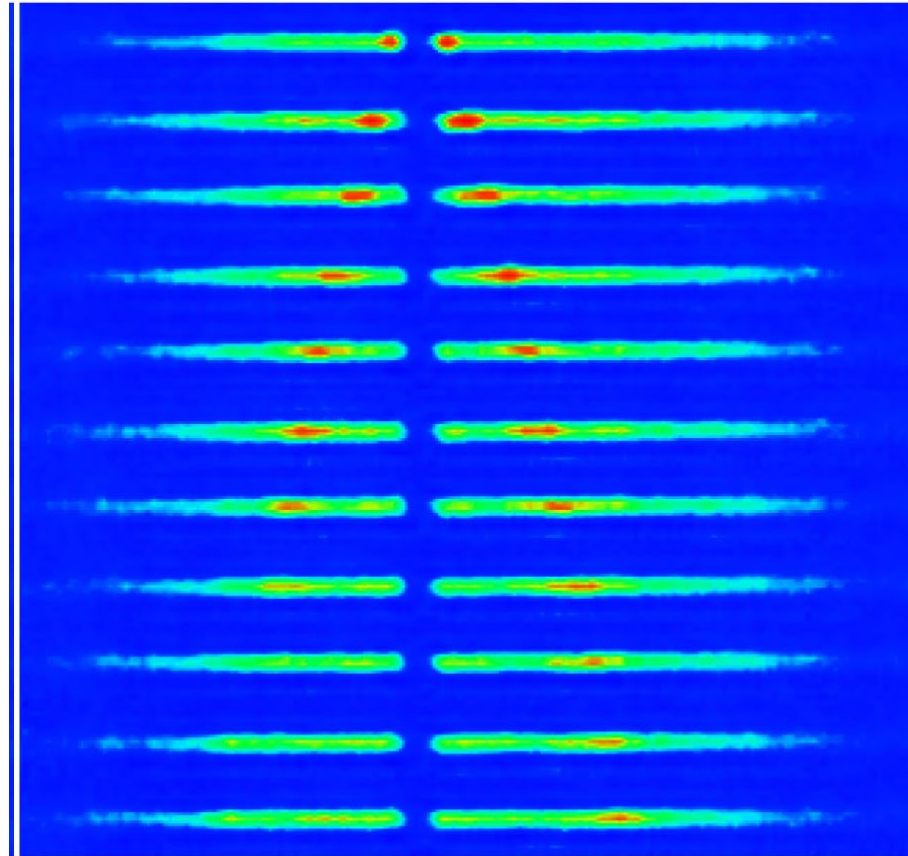


Lecture 15: Dynamics of Quantum Gas

A wide-angle photograph of the Chicago skyline at night, viewed from across a body of water. The city lights are reflected in the water. A white diamond shape is drawn on the right side of the skyline, highlighting a specific building.

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Sound propagation



0.5 mm

Sound waves

$$i\hbar\partial_t\psi = \left(\frac{\hbar^2k^2}{2m} + g|\psi|^2\right)\psi$$

Ground state $i\hbar\partial_t\psi_0 = \left(\frac{\hbar^2k^2}{2m} + g|\psi_0|^2\right)\psi_0 = \mu_0\psi_0$

Perturbation $\psi = \psi_0 + \epsilon\psi_1$ with $\epsilon \ll 1$ linearize the equation

Substitution gives $-i\hbar\partial_t\psi_1 = \left(\frac{\hbar^2k^2}{2m} + 2g|\psi_0|^2\right)\psi_1 + g\psi_0^2\psi_1^*$

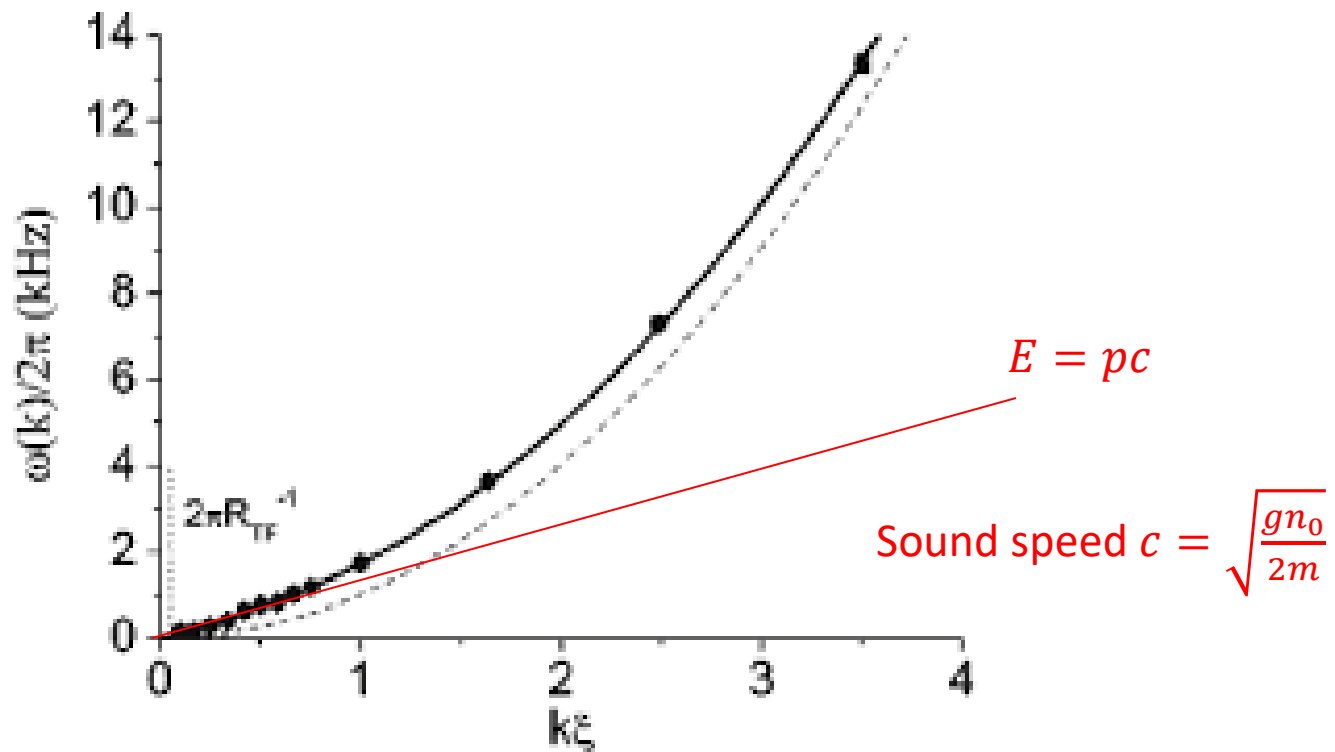
Ansatz: $\psi_1 = ue^{i(kx-\omega t)} + ve^{-i(kx-\omega t)}$

We get

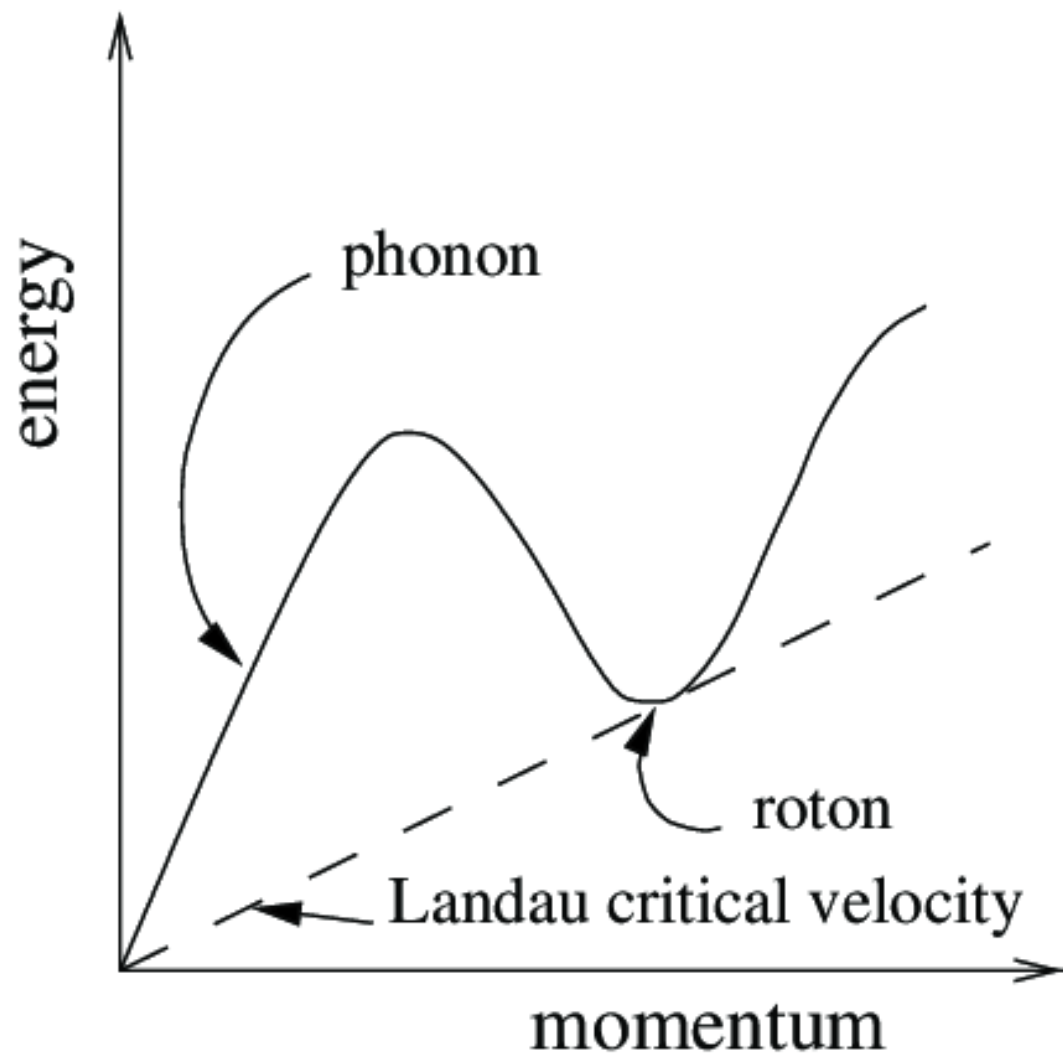
$$\begin{aligned} \left(\frac{\hbar^2k^2}{2m} + 2gn_0 - \hbar\omega\right)u - gn_0v &= 0 \\ \left(\frac{\hbar^2k^2}{2m} + 2gn_0 + \hbar\omega\right)v - gn_0u &= 0 \end{aligned}$$

Solution exists when $\begin{vmatrix} \frac{\hbar^2k^2}{2m} + 2gn_0 - \hbar\omega & -gn_0 \\ -gn_0 & \frac{\hbar^2k^2}{2m} + 2gn_0 + \hbar\omega \end{vmatrix} = 0$

Bogoliubov dispersion $E = \sqrt{\frac{p^2}{2m} \left(\frac{p^2}{2m} + gn_0 \right)}$

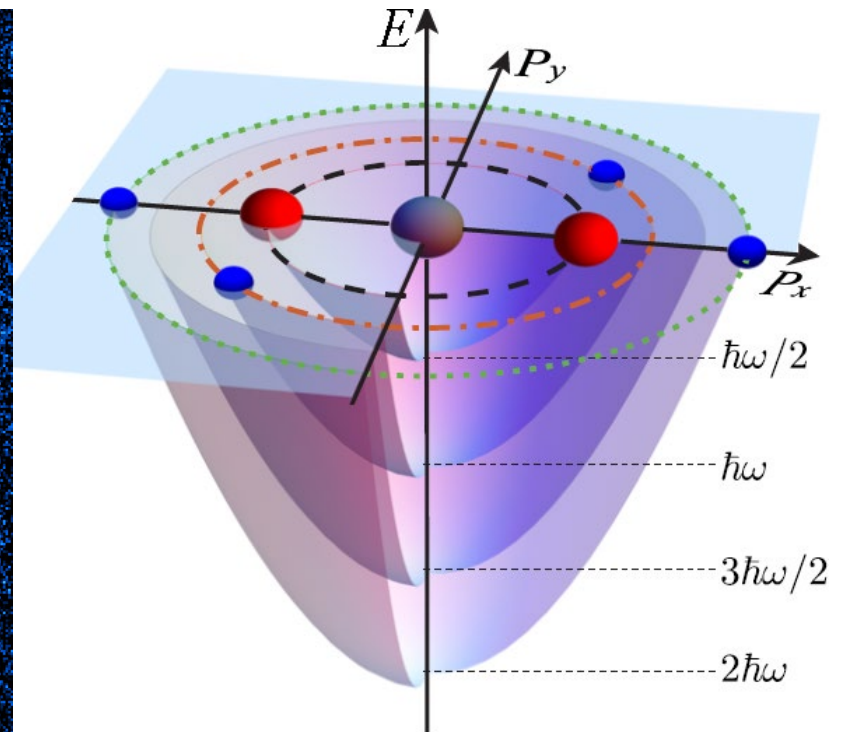
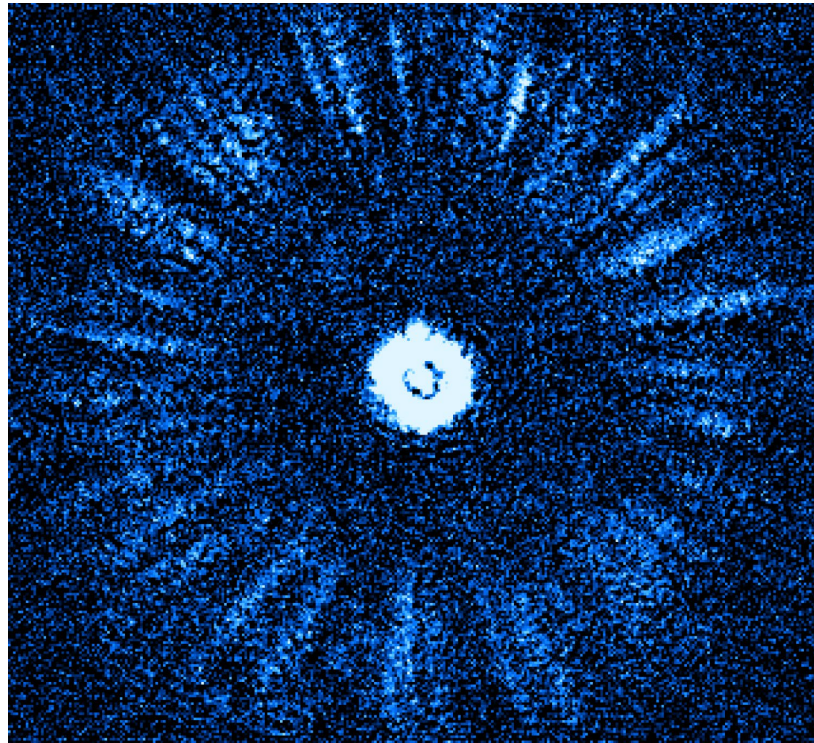


Superfluidity





Bose fireworks



Collective emission of matter-wave jets from driven Bose-Einstein condensates, Nature 551 (2017)