2021P238 Midterm 11/4/2021

1A. π pulse $\Omega t/2 = \pi/2 \Rightarrow t = \pi/\Omega \Rightarrow U_\pi = -\frac{i}{\Omega}\begin{pmatrix} 0 & -\Omega \\ \Omega & 0 \end{pmatrix} = -i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\pi/2$ $= \pi/4 \Rightarrow t = \pi/2\Omega \Rightarrow U_{\pi/2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$

B $U_{free} = \mathbb{1}$ when $\Delta = 0$

$U = U_{\pi/2} U_\pi U_{\pi/2} = U_{2\pi} = -\mathbb{1}$

C. $U_{free} = \begin{pmatrix} e^{-i\Delta t/2} & 0 \\ 0 & e^{i\Delta t/2} \end{pmatrix} \equiv \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix}$

$$U = \frac{-i}{2}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

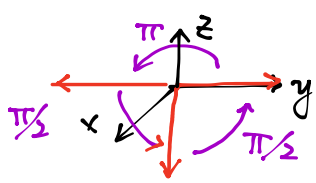
$$= -\frac{i}{2}\begin{pmatrix} e^{-i\phi} & -ie^{i\phi} \\ -ie^{-i\phi} & e^{i\phi} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} e^{-i\phi} & -ie^{-i\phi} \\ -ie^{i\phi} & e^{i\phi} \end{pmatrix}$$

$$= -\frac{i}{2}\begin{pmatrix} e^{-i\phi} & -ie^{i\phi} \\ -ie^{-i\phi} & e^{i\phi} \end{pmatrix}\begin{pmatrix} -ie^{i\phi} & e^{i\phi} \\ e^{-i\phi} & -ie^{-i\phi} \end{pmatrix}$$

$$= -\frac{i}{2}\begin{pmatrix} -i-i & 1-1 \\ -1+1 & -i-i \end{pmatrix} = -\frac{i}{2}(-2i)\,\mathbb{1} = -\mathbb{1}$$

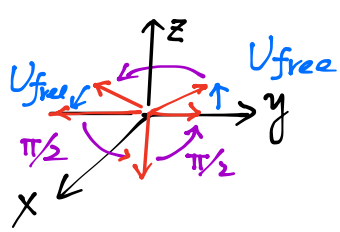
The echo pulse eliminates the phase shift during free evolution.

d. $\Delta = 0$



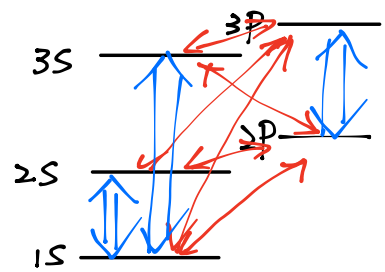
$\pi/2$, π pulses rotate Bloch vector along x-axis.

$\Delta \neq 0$



U_{free} rotates along Bloch vector along z axis.

2. A see right figure.



B. $H_2 = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega b^+ b + \frac{\hbar g_2}{2}(\sigma^+ b b + \sigma^- b^+ b^+)$

C. $|g,0\rangle$ gives Energy $-\frac{\hbar\omega_0}{2}$

$|g,1\rangle$ gives Energy $= -\frac{\hbar\omega_0}{2} + \hbar\omega = 0.$

$|g,2\rangle$ couples to $|e,0\rangle$

$$H_2 = \frac{1}{2}\hbar\omega_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\hbar g_2 \begin{pmatrix} 0 & \sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}$$

$$= \frac{\hbar}{2}(\omega_0 \hat{I} + \sqrt{2} g_2 \sigma_x)$$

$$\text{Energy} = \frac{\hbar}{2}\omega_0 \pm \frac{\hbar}{2}\sqrt{2} g_2$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1$$