

Class website <https://ultracold.uchicago.edu/node/189>

The screenshot shows a web browser displaying the Chin Lab website. The URL in the address bar is <https://ultracold.uchicago.edu/>. The page title is "Chin Lab at the University of Chicago" with the subtitle "Ultracold atomic and molecular physics". A sidebar on the left contains links to "Home", "Publications", "News", "Ultracold gallery", "Group Videos", "Research", "People/Contact", "Courses/Outreach", "Open Positions", "Collaborative project", and "Internal". A "Recent Updates" section lists "Group meeting presentations" and "Google Sheet of COVID 19 research resumption schedule". The main content area is titled "Autumn 2021 P238" and describes the course "Physics 23800 Atomic Physics". It provides details about the day/time (TuTh 11:00am - 12:20pm), location (KPTC 309), lecturer (Cheng Chin, cchin@uchicago.edu), office hours (TuTh 1 - 2 pm on Zoom), grader (Shu Nagata, nagatashu@uchicago.edu), and class outline. Below this, sections for Q1, Q2, Q3, and Q4 provide notes and lecture outlines. At the bottom, there are links to "Textbook (Recommended) Atomic Physics by Christopher J. Foot, Oxford, 2005", "Evaluation" (problem sets, midterms, discussions), and "Homeworks and midterms" (each homework contains 2-3 questions, late policy: -10% per day). Navigation links include "Physics Courses", "up", "Spring 2021 P334", and "Log in to post comments".

What are atoms good for : Precision Measurements

Quantum control

Explore new quantum physics

Quantum information processing

Lecture I Precision Measurements with atoms

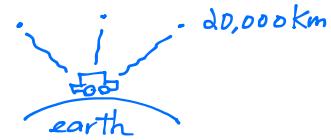
Key words: Atomic clocks, fine and hyperfine structure, parity non-conserving interaction, LIGO.

Why are atoms good for precision measurements?

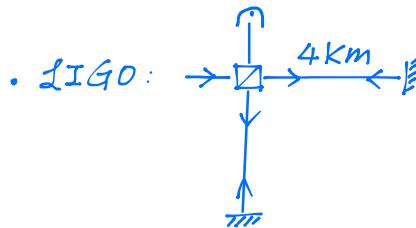
- Atomic energy levels are well defined.
- Atoms are identical particles.
- Easy to prepare in different states with lasers.

How precise do we need?

- GPS: trilateration from few satellites.



$$\begin{aligned} \text{Precision } \Theta \text{ Im} &\Rightarrow t = L/c \\ &\Rightarrow \Delta t = \Delta L/c \\ &\Rightarrow \Delta t/t = \Delta L/L = 1/2 \times 10^7 = 5 \times 10^{-8} \\ &\Rightarrow 8\text{-digit measurement.} \end{aligned}$$



$$\begin{aligned} \Delta L/L &= 10^{-19} && 19 \text{ digits.} \\ \Rightarrow \Delta L &= 4 \times 10^{-10} \mu\text{m} \end{aligned}$$

- Atomic clock $\Delta f/f = 10^{-18}$

- EDM exp. (D. DeMille)

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Published: 22 January 2014
An optical lattice clock with accuracy and stability at the 10^{-18} level
B. J. Bloom, T. L. Nicholson, J. R. Williams, S. L. Campbell, M. Bishof, X. Zhang, W. Zhang, S. L. Bromley & J. D. Thompson

Nature 506, 71–75 (2014) | [Cite this article](#)
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Abstract
Progress in atomic, optical and quantum science^{1,2} has led to rapid improvements in atomic clocks. At the same time, atomic clock research has helped to advance the frontiers of science, affecting both fundamental and applied research. The ability to control quantum states of individual atoms and photons is central to quantum information science and precision measurement. In this work, we demonstrate a many-atom system that achieves systematic uncertainty in frequency standard^{3–5}. Although many atom-lattice clocks have shown advantages in measurement precision over trapped-ion clocks^{6,7}, their accuracy has remained ‘at the worse’^{8,9,10}. Here we demonstrate a many-atom system that achieves an accuracy of 6.4×10^{-18} , which is better than a single-ion clock, but also reduces the required measurement time by two orders of magnitude. By systematically evaluating all known sources of uncertainty, including *in situ* monitoring of the blackbody radiation environment, we improve the accuracy of optical lattice clocks by a factor of 22. This single clock has simultaneously achieved the best known performance in the key characteristics necessary for consideration as a primary time and frequency standard.

Editorial Summary
Editor’s choice: An optical lattice clock with accuracy and stability at the 10^{-18} level
Whether for the advancement of science, solving the laws of physics or for applications to the benefit of society, atomic clocks have more stability and more accuracy than atomic clock ‘worse’ counterparts have achieved before.

Sections | **Figures** | **References**

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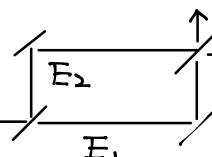
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How is it possible that one can measure something to $10 \sim 19$ digits?

Atom interferometry (idea from laser interferometry)

photons: 

$$E = E_1 e^{i k L_1} + E_2 e^{i k L_2}$$
$$I = |E|^2 \cos^2 K(L_1 - L_2)$$

(Assume $E_1 \approx E_2$)

- atoms
- $|e\rangle$ excited state energy $E_e \equiv \hbar\omega_0$ $|e\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
 - $|g\rangle$ ground state energy $E_g \equiv 0$ $|g\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

{ Schrödinger's
Equation } $i\hbar\dot{\varphi} = \hbar\omega_0 \Rightarrow \varphi_{it} = e^{-i\omega_0 t} \varphi_{t=0}$

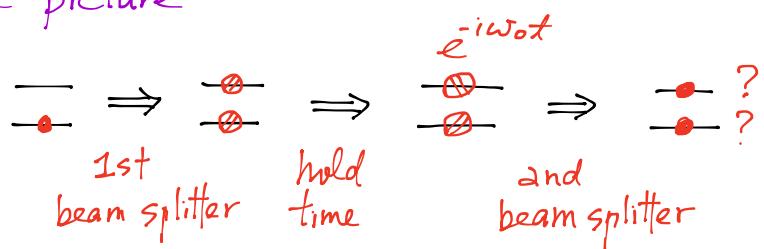
Step 1: prepare atoms in the g state $|g\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Step 2: make a superposition state $\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Step 3: wait for $t \Rightarrow \frac{1}{\sqrt{2}}(|g\rangle + e^{-i\omega_0 t}|e\rangle)$ (first beam splitter)

Step 4: apply the 2nd beam splitter $\Rightarrow ??$ Details see next slide.

Physical picture



Lecture 2 Radiative Process and Rabi flopping

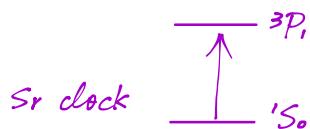
9/30/2021

(Foot: 7.1 and 7.3.1)

Radiative excitation



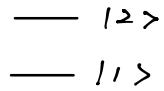
Energy separation $E_4 - E_3 = h \times 9192631770 \text{ Hz}$



Energy separation = $h \times 429228066418007 \text{ Hz}$

$$i\hbar \partial_t \varphi = H\varphi \quad \text{If } H = \hbar\omega_0 = \text{const.} \quad i\partial_t \varphi = \omega_0 \varphi \Rightarrow \varphi(t) = \varphi(0) e^{-i\omega_0 t}$$

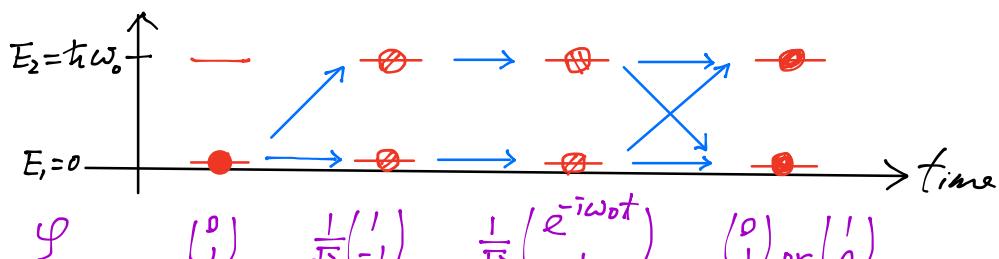
Now consider a ω -level system



$$\varphi(t) = C_1(t)|1\rangle + C_2(t)|2\rangle$$

$$H = \hbar \begin{bmatrix} E_2 & 0 \\ 0 & E_1 \end{bmatrix} \Rightarrow C_j(t) = C_j(0) e^{-i\omega_j t} \quad j=1..2.$$

Ramsey spectroscopy



step preparation free evolution detection

beam splitting
($\pi/2$ pulse)

beam splitting or
beam combining
($\pi/2$ pulse)

$$\left\{ \pi/2 \text{ pulse} \equiv U(\pi/2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \right.$$

$$\left. \text{In general } U(\theta) = \begin{pmatrix} \cos \theta/2 & \sin \theta/2 \\ -\sin \theta/2 & \cos \theta/2 \end{pmatrix} \right.$$

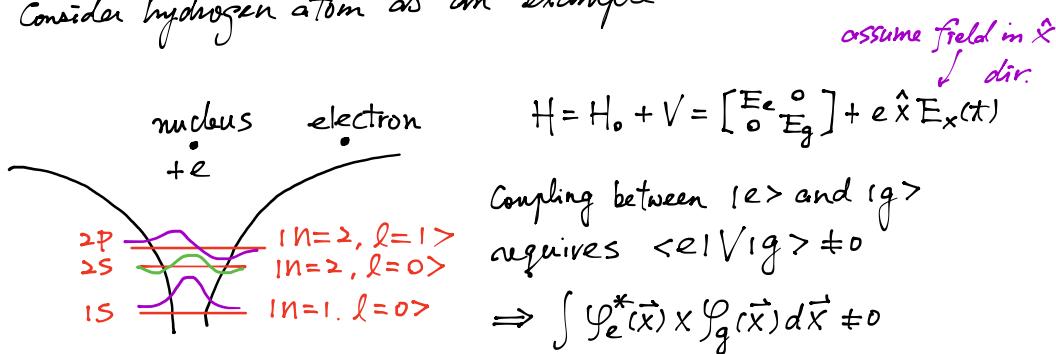
Show that $U = e^{-i\sigma_x \theta/2}$, $U(\theta+\phi) = U(\theta)U(\phi)$, $U(\pi) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $U(2\pi) = -I$

\Rightarrow The only thing we need to do is to implement a rotation operation, which is the transition between $|g\rangle$ & $|e\rangle$.

How do we induce a transition?

- Laser transition (Electric dipole transition, interaction $V = -d \cdot E$) \nexists I tran.

Consider hydrogen atom as an example



$\Rightarrow E_z$ transition only couples states with opposite parity.

$$|1S\rangle \leftrightarrow |2P\rangle \quad \checkmark$$

$$|1S\rangle \leftrightarrow |2S\rangle \quad \times$$

$$\Rightarrow H = \hbar \begin{bmatrix} \omega_0 & \Omega \\ -\Omega^* & 0 \end{bmatrix}, \quad \hbar \omega_0 = E_e - E_g, \quad \hbar \Omega = \langle e | V | g \rangle \equiv - \langle d | E \rangle$$

\uparrow
resonance freq. $\hbar \Omega^* = \langle g | V | e \rangle$

For a time-dependent laser field $E(t) = E \cos \omega t$, we write

$$H = \hbar \omega_0 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \hbar \Omega \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cos \omega t \quad . \quad \text{assume } \Omega \in \mathbb{R}$$

\uparrow
Rabi freq.

- Microwave transition (Magnetic dipole transition, int. $V = -\mu \cdot B$)

$$V = -\vec{\mu} \cdot \vec{B} = -g \vec{L} \cdot \vec{B} = -g \mu_B \frac{\vec{L}}{\hbar} \cdot \vec{B}$$

(MI transition)

γ : gyro-magnetic ratio

g : g -factor

L : angular momentum

μ_B : Bohr magneton

In general, \vec{L} comes from e^- as $\vec{L} = \hbar(\vec{l} + \vec{s})$. orbital & spin angular moment.

$g=1$ for orbital. $g \approx 2$ for spin.

There is also nucleus spin contribution $\vec{L} = \hbar \vec{i}$, which is much weaker

Cs atomic clock is based on microwave transition

$$\langle e | V | g \rangle = -\mu_B \langle g \vec{L} \cdot \vec{B} \rangle \equiv \hbar \Omega \cos \omega t$$

$$\text{We arrive at the same } H = \hbar \omega_0 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \hbar \Omega \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cos \omega t$$

Our next goal is to solve

Schroedinger's eqn with the above Hamiltonian.

$$i\hbar \partial_t \psi = H(t) \psi \quad \psi = \begin{pmatrix} C_2 \\ C_1 \end{pmatrix} \quad |C_1|^2 + |C_2|^2 = 1$$

Unfortunately, no analytic solution... we will introduce 2 methods.

1. Perturbation. assume light is weak.

0th order: light is off \Rightarrow atoms in the ground state. $\psi(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\text{1st order: } i\partial_t \begin{pmatrix} C_2 \\ C_1 \end{pmatrix} = \left[\omega_0 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \Omega \cos \omega t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right] \begin{pmatrix} C_2 \\ C_1 \end{pmatrix}$$

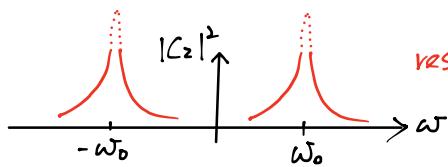
$$\begin{aligned} iC'_2 &= \omega_0 C_2 + \Omega \cos \omega t C_1 \\ &\approx \omega_0 C_2 + \Omega \cos \omega t \end{aligned}$$

$$\begin{aligned} \text{let } C_2 &= A e^{i\omega t} + B e^{-i\omega t} & -\omega A e^{i\omega t} + \omega B e^{-i\omega t} \\ &= \omega_0 A e^{i\omega t} + \omega_0 B e^{-i\omega t} + \frac{\Omega}{2} e^{i\omega t} + \frac{\Omega}{2} e^{-i\omega t} \end{aligned}$$

$$\Rightarrow (\omega_0 + \omega) A = \frac{\Omega}{2}, (\omega - \omega_0) B = \frac{\Omega}{2}$$

$$\Rightarrow A = \frac{\Omega}{\omega(\omega + \omega_0)} . B = \frac{\Omega^2}{\omega(\omega - \omega_0)} \gg A . |C_2|^2 \approx \frac{\Omega^2}{4(\omega - \omega_0)^2}$$

↑ near resonance detuning
 $\Delta = \omega - \omega_0 \rightarrow 0$



resonance occurs when $\Delta = 0$

2. Rotating frame & rotating wave approximation (RWA)

Idea: go to the frame that corotates with the laser.

$$i\dot{\phi} = \frac{\omega_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \phi + \Omega \cos \omega t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \phi$$

$$\text{let } \phi = \begin{pmatrix} e^{-i\omega t/2} & 0 \\ 0 & e^{i\omega t/2} \end{pmatrix} \hat{\phi} \equiv \hat{R} \phi$$

$$\Rightarrow \text{L.H.S. } i\dot{\phi} = \frac{\omega}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \hat{R} \phi + \hat{R} i\dot{\phi}$$

$$\text{R.H.S. } \frac{\omega_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \hat{R} \phi + \Omega \cos \omega t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \hat{R} \phi$$

Multiply both sides by $R' = \begin{pmatrix} e^{i\omega t/2} & 0 \\ 0 & e^{-i\omega t/2} \end{pmatrix}$, we get

$$i\dot{\phi} = \frac{\Delta}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \phi + \underline{\Omega \cos \omega t R' \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} R \phi}$$

$$= \frac{1}{2} (e^{i\omega t} + e^{-i\omega t}) \begin{pmatrix} 0 & e^{i\omega t} \\ e^{-i\omega t} & 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & e^{2i\omega t} \\ e^{-2i\omega t} & 0 \end{pmatrix}$$

This is the rotating wave approximation

$$\Rightarrow i\dot{\phi} = -\frac{\Delta}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \phi + \frac{\Omega}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \phi$$

equivalent to replace $\cos \omega t$ by $e^{-i\omega t}$

Now we can solve it analytically.

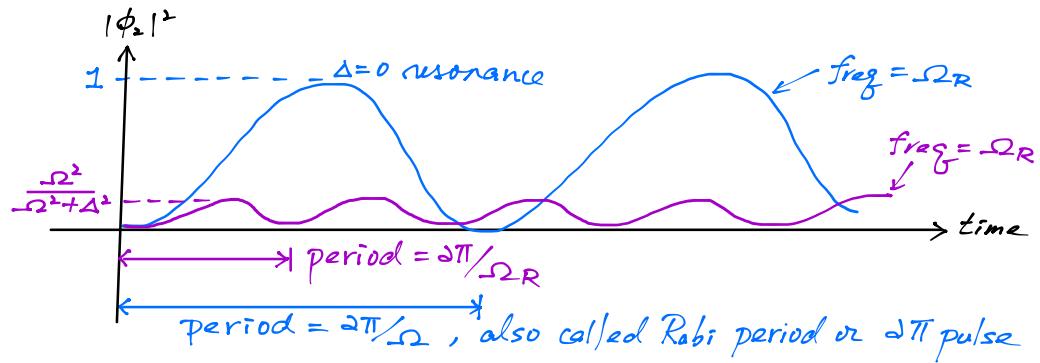
Assume $\phi = \begin{pmatrix} \phi_2 \\ \phi_1 \end{pmatrix}$ $\phi_1(t=0)=1, \phi_2(t=0)=0$ we get

$$\Rightarrow |C_2(t)|^2 = \frac{\Omega^2}{\Omega_R^2} \sin^2 \Omega_R \frac{t}{2}, \quad \Omega_R = \sqrt{\Omega^2 + \Delta^2}. \text{ Generalized Rabi frag.}$$

More generally, given initial state $\phi(0) \Rightarrow \phi(t) = \hat{U} \phi(0)$,
 evolution operator $\hat{U}(t) = e^{-iHt/\hbar}$

Derive the form of $\hat{U}(t)$ in the rotating & Lab frame.

$$\text{Plot the solution } |\phi_2|^2 = \frac{\Omega^2}{\Omega_R^2 + \Delta^2} \sin^2 \frac{\Omega_R t}{2}$$

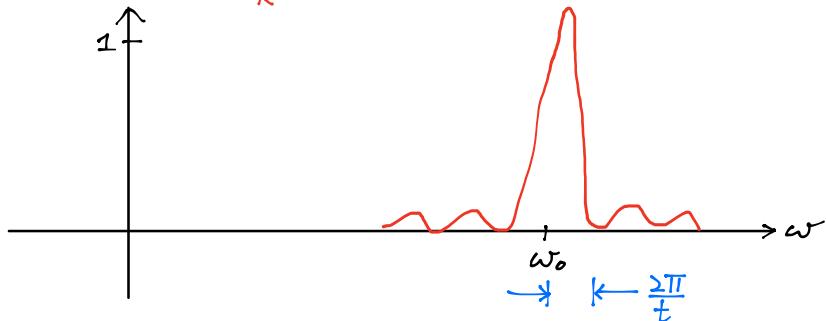


- This is called Rabi flopping
- 100% oscillation occurs on resonance
- A resonant pulse that lasts for $2\pi/\Omega$ is called a 2π pulse
- A resonant pulse that lasts for π/Ω is called a π pulse
- A resonant pulse that lasts for $\pi/2\Omega$ is called a $\pi/2$ pulse

Consider max transfer (π pulse) on resonance $t = \pi/\Omega$. we have

Rabi Spectroscopy

$$|\phi_2|^2 = \frac{\Omega^2}{\Omega_R^2} \sin^2 \frac{\pi \Omega_R t}{\Omega}$$



Frequency resolution $\Delta\omega \approx 2\pi/At$
↑ pulse width

$$\Rightarrow \text{energy resolution } \Delta E = \hbar \Delta\omega = \hbar \frac{2\pi}{At} \Rightarrow \Delta E At = \hbar$$

consistent with Heisenberg's uncertainty principle.