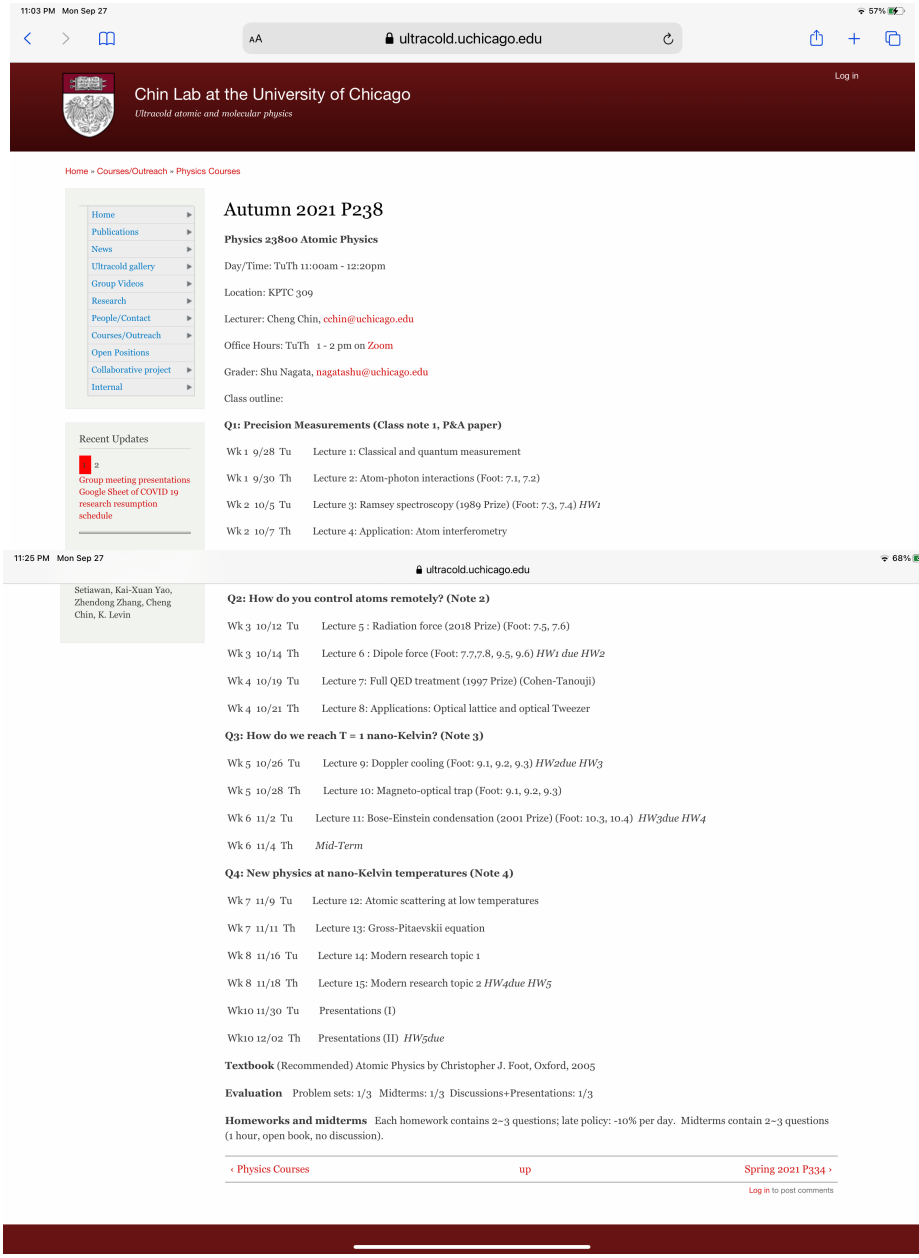
Class website https://ultracold.uchicago.edu/node/189



What are atoms good for: Precision Measurements

Quantum control

Explore new quantum physics

Quantum information processing

Lecture 1. Precision Measurements with atoms

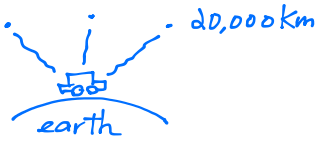
Key words: Atomic clocks, fine and hyperfine structure, parity non-conserving interaction, LIGO.

Why are atoms good for precision measurements?

- Atomic energy levels are well defined.
- Atoms are identical particles.
- Easy to prepare in different states with lasers.

How precise do we need?

- GPS: trilateration from few satelites.

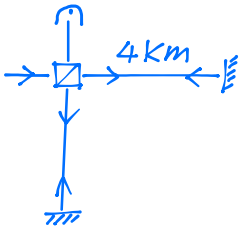


Precision @ 1m $\Rightarrow t = L/c$

$\Rightarrow \Delta t = \Delta L/c$

$\Rightarrow \Delta t/t = \Delta L/L = 1/2\times10^{7} = 5\times10^{-8}$

$\Rightarrow$ 8-digit measurement.

- LIGO: 

$\Delta L/L = 10^{-19}$ 19 digits.

$\Rightarrow \Delta L = 4\times10^{-10}\,\mu m$

- Atomic clock $\Delta f/f = 10^{-18}$

nature

An optical lattice clock with accuracy and stability at the 10^{-18} level

Published: 22 January 2014

Abstract

Editorial Summary

- EDM exp. (D. DeMille)

nature

An optical lattice clock with accuracy and stability at the 10^{-18} level

Published: 22 January 2014

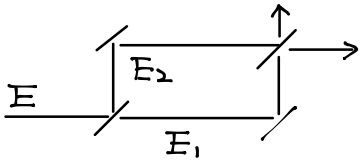
Abstract

Editorial Summary

How is it possible that one can measure something to 10~19 digits?

Atom interferometry (idea from laser interferometry)

photons:



$E = E_1 e^{ikL_1} + E_2 e^{ikL_2}$

$I = |E|^2 \cos^2 K(L_1 - L_2)$

(Assume $E_1 \simeq E_2$)

atoms

—— $|e\rangle$ excited state. energy $E_e \equiv \hbar\omega_0$ $|e\rangle \equiv \begin{pmatrix} e^{-i\omega_0 t} \\ 0 \end{pmatrix}$

—— $|g\rangle$ ground state. energy $E_g \equiv 0$ $|g\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

{ Schroedinger's Equation $i\hbar\dot{\varphi} = \hbar\omega_0 \Rightarrow \varphi(t) = e^{-i\omega_0 t}\varphi(0)$

Step 1: prepare atoms in the g state $|g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

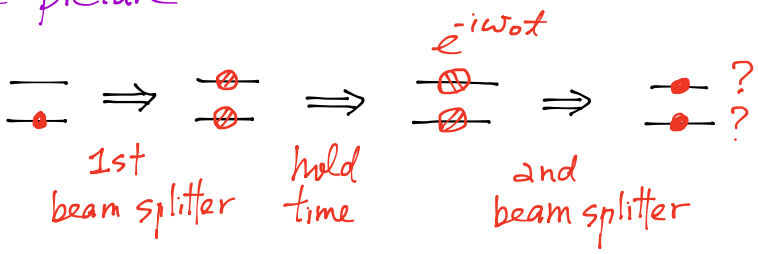
Step 2: make a superposition state $\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (first beam splitter)

Step 3: wait for $t \Rightarrow \frac{1}{\sqrt{2}}(|g\rangle + e^{-i\omega_0 t}|e\rangle)$

Step 4: apply the 2nd beam splitter $\Rightarrow$??

Details see next slide.

Physical picture



Lecture 2 Radiative Process and Rabi Flopping

9/30/2021

(Foot: 7.1 and 7.3.1)

Radiative excitation

Cs clock

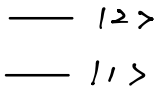
Energy separation $E_4 - E_3 = h \times 9192631770\ \text{Hz}$

Sr clock

Energy separation $= h \times 429228066418007\ \text{Hz}$

$i\hbar \partial_t \psi = H\psi$ If $H = \hbar\omega_0 = \text{const.}$ $i\partial_t \psi = \omega_0 \psi \Rightarrow \psi(t) = \psi(0) e^{-i\omega t}$

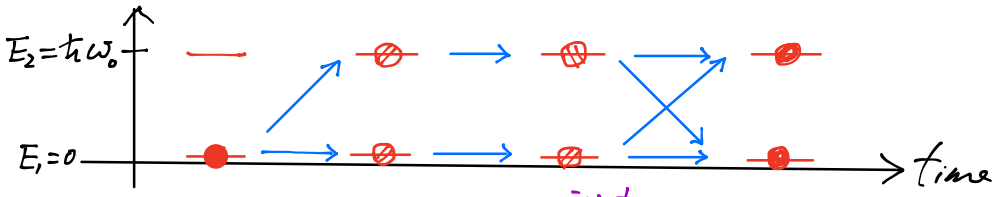
Now consider a 2-level system



$\psi(t) = C_1(t)|1\rangle + C_2(t)|2\rangle$

$H = \hbar \begin{bmatrix} E_2 & 0 \\ 0 & E_1 \end{bmatrix} \Rightarrow C_j(t) = C_j(0) e^{-i\omega_j t}$ $j = 1, 2.$

Ramsey spectroscopy



ψ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\omega_0 t} \\ -1 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

step: preparation — free evolution — detection

beam splitting ($\pi/2$ pulse)

beam splitting or beam combining ($\pi/2$ pulse)

$\pi/2$ pulse $\equiv U(\pi/2) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

In general $U(\theta) = \begin{pmatrix} \cos\theta/2 & \sin\theta/2 \\ -\sin\theta/2 & \cos\theta/2 \end{pmatrix}$

Show that $U = e^{-i\sigma_x \theta/2}$, $U(\theta+\phi) = U(\theta)U(\phi)$, $U(\pi) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $U(2\pi) = -1$

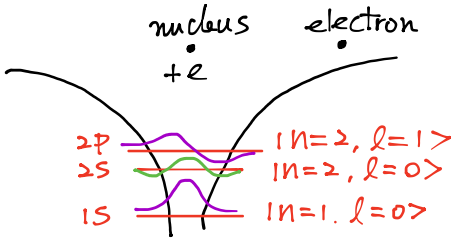
⇒ The only thing we need to do is to implement a rotation operation, which is the transition between $|g\rangle$ & $|e\rangle$.

How do we induce a transition?

- Laser transition (Electric dipole transition, interaction $V = -d \cdot E$) $\Rightarrow$ E1 tran.

Consider hydrogen atom as an example

assume field in $\hat{x}$ dir.



$$H = H_0 + V = \begin{bmatrix} E_e & 0 \\ 0 & E_g \end{bmatrix} + e\hat{x}E_x(t)$$

Coupling between $|e\rangle$ and $|g\rangle$ requires $\langle e|V|g\rangle \neq 0$

$$\Rightarrow \int \varphi_e^*(\vec{x}) \times \varphi_g(\vec{x}) d\vec{x} \neq 0$$

⇒ E1 transition only couples states with opposite parity.

$|1s\rangle \longleftrightarrow |2p\rangle$ ✓

$|1s\rangle \longleftrightarrow |2s\rangle$ ×

$$\Rightarrow H = \hbar \begin{bmatrix} \omega_0 & \Omega \\ \Omega^* & 0 \end{bmatrix}, \quad \hbar\omega_0 = E_e - E_g, \quad \hbar\Omega = \langle e|V|g\rangle \equiv -\langle d\rangle E$$

ω_0: resonance freq

$$\hbar\Omega^* = \langle g|V|e\rangle$$

For a time-dependent laser field $E(t) = E\cos\omega t$, we write

$$H = \hbar\omega_0 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \hbar\Omega \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cos\omega t$$

Ω: Rabi freq.

assume $\Omega \in \mathbb{R}$.

• Microwave transition (Magnetic dipole transition, int. $V=-\mu\cdot B$)

$V=-\vec{\mu}\cdot\vec{B}=-\gamma\vec{L}\cdot\vec{B}=-g\mu_B\frac{\vec{L}}{\hbar}\cdot\vec{B}$

(M1 transition)

γ: gyro-magnetic ratio

g: g-factor

L: angular momentum

μ_B: Bohr magneton

In general, $\vec{L}$ comes from e^- as $\vec{L}=\hbar(\vec{l}+\vec{s})$. orbital & spin angular momen.

$g=1$ for orbital. $g\approx 2$ for spin.

There is also nucleus spin contribution $\vec{L}=\hbar\vec{i}$, which is much weaker

Cs atomic clock is based on μwave transition

$\langle e|V|g\rangle=-\mu_B\langle g\vec{L}\cdot\vec{B}\rangle\equiv\hbar\Omega\cos\omega t$

We arrive at the same $H=\hbar\omega_0\begin{bmatrix}1&0\\0&0\end{bmatrix}+\hbar\Omega\begin{bmatrix}0&1\\1&0\end{bmatrix}\cos\omega t$

Our next goal is to solve

Schroedinger's eqn with the above Hamiltonian.

$i\hbar\partial_t\Psi=H(t)\Psi\qquad\Psi=\begin{pmatrix}c_2\\c_1\end{pmatrix}\qquad|c_1|^2+|c_2|^2=1$

Unfortunately, no analytic solution... We will introduce 2 methods.

1. Perturbation. assume light is weak.

0th order: light is off ⇒ atoms in the ground state. $\Psi(0)=\begin{pmatrix}0\\1\end{pmatrix}$

1st order: $i\partial_t\begin{pmatrix}c_2\\c_1\end{pmatrix}=\left[\omega_0\begin{pmatrix}1&0\\0&0\end{pmatrix}+\Omega\cos\omega t\begin{pmatrix}0&1\\1&0\end{pmatrix}\right]\begin{pmatrix}c_2\\c_1\end{pmatrix}$

$ic_2'=\omega_0 c_2+\Omega\cos\omega t\, c_1$

$\approx\omega_0 c_2+\Omega\cos\omega t$

let $c_2=Ae^{i\omega t}+Be^{-i\omega t}$

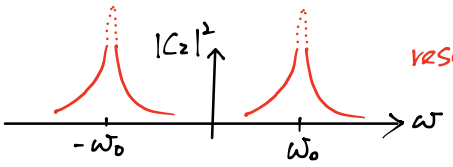
$-\omega Ae^{i\omega t}+\omega Be^{-i\omega t}$

$=\omega_0 Ae^{i\omega t}+\omega_0 Be^{-i\omega t}+\frac{\Omega}{2}e^{i\omega t}+\frac{\Omega}{2}e^{-i\omega t}$

$\Rightarrow(\omega_0+\omega)A=\frac{\Omega}{2}\quad(\omega-\omega_0)B=\frac{\Omega}{2}$

$$\Rightarrow A = \frac{\Omega}{2(\omega+\omega_0)} \quad B = \frac{\Omega}{2(\omega-\omega_0)} \gg A \quad |C_2|^2 \approx \frac{\Omega^2}{4(\omega-\omega_0)^2}$$

↖ near resonance detunning $\Delta = \omega - \omega_0 \to 0$



resonance occurs when $\Delta = 0$

3. Rotating frame & rotating wave approximation (RWA)

Idea: go to the frame that corotates with the laser.

$$i\partial_t \varphi = \frac{\omega_0}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\varphi + \Omega\cos\omega t \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\varphi$$

let $\varphi = \begin{pmatrix} e^{-i\omega t/2} & 0 \\ 0 & e^{i\omega t/2}\end{pmatrix}\phi \equiv \hat{R}\phi$

$$\Rightarrow \quad \text{L.H.S. } i\partial_t\varphi = \frac{\omega}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\hat{R}\phi + \hat{R}\, i\partial_t\varphi$$

$$\text{R.H.S. } \frac{\omega_0}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\hat{R}\phi + \Omega\cos\omega t\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\hat{R}\phi$$

Multiply both sides by $R^{-1} = \begin{pmatrix} e^{i\omega t/2} & 0 \\ 0 & e^{-i\omega t/2}\end{pmatrix}$, we get

$$i\partial_t\phi = \frac{-\Delta}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\phi + \Omega\cos\omega t R^{-1}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}R\phi$$

$$= \frac{1}{2}(e^{i\omega t} + e^{-i\omega t})\begin{pmatrix}0 & e^{i\omega t}\\ e^{-i\omega t} & 0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0 & e^{2i\omega t}\\ e^{-2i\omega t} & 0\end{pmatrix}$$

0

This is the rotating wave approximation

equivalent to replace $\cos\omega t$ by $e^{-i\omega t}$

$$\Rightarrow i\partial_t\phi = -\frac{\Delta}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\phi + \frac{\Omega}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\phi$$

Now we can solve it analytically.

Assume $\phi = \begin{pmatrix}\phi_2\\ \phi_1\end{pmatrix}$ $\phi_1(t=0) = 1$, $\phi_2(t=0) = 0$ we get

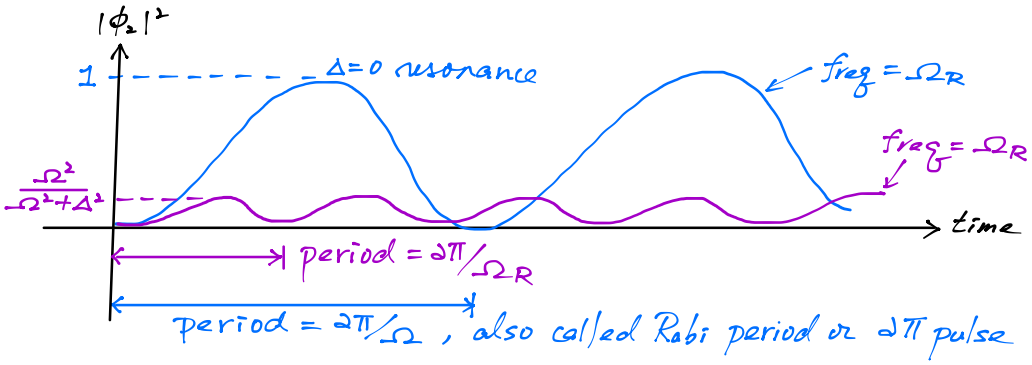
$$\Rightarrow |C_2(t)|^2 = \frac{\Omega^2}{\Omega_R^2}\sin^2\Omega_R\frac{t}{2}. \quad \Omega_R = \sqrt{\Omega^2+\Delta^2}: \text{ Generalized Robi freq.}$$

More generally, given initial state $\phi(0) \Rightarrow \phi(t) = \hat{U}\phi(0)$,

evolution operator $\hat{U}(t) = e^{-iHt/\hbar}$

Derive the form of $\hat{U}(t)$ in the rotating & Lab frame.

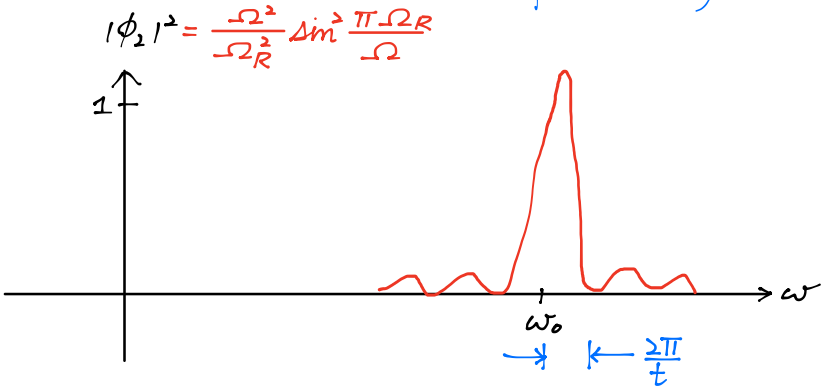
Plot the solution $|\phi_2|^2 = \frac{\Omega^2}{\Omega_R^2} \sin^2 \frac{\Omega_R t}{2}$



- This is called Rabi flopping
- 100% oscillation occurs on resonance
- A resonant pulse that lasts for $2\pi/\Omega$ is called a 2π pulse
- A resonant pulse that lasts for π/Ω is called a π pulse
- A resonant pulse that lasts for $\pi/2\Omega$ is called a $\pi/2$ pulse

Consider max transfer (π pulse) on resonance $t = \pi/\Omega$. We have

Rabi Spectroscopy



Frequency resolution $\Delta\omega \approx 2\pi/\Delta t$ ← pulse width

$\Rightarrow$ energy resolution $\Delta E = \hbar \Delta\omega = \hbar \frac{2\pi}{\Delta t} \Rightarrow \Delta E \Delta t = h$

consistent with Heisenberg's uncertainty principle.