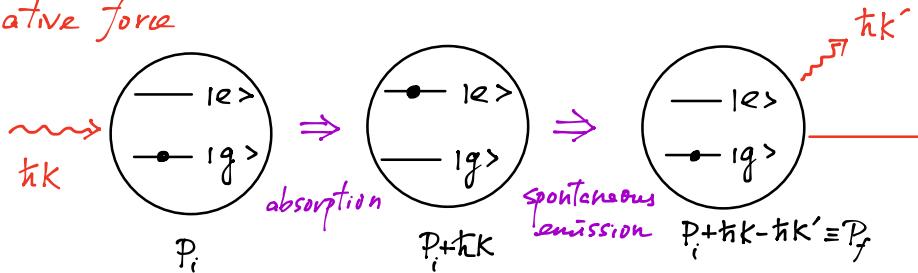


Control atoms remotely with light \Rightarrow laser cooling and trapping.

Radiative force



$$\text{Momentum conservation: } p_i + \hbar k = P = p_f + \hbar k'$$

$$\text{Energy conservation: } \hbar\omega + p_i^2/2m = \hbar\omega_0 + p^2/2m = \hbar\omega' + p_f^2/2m$$

Resonance condition

$$\text{Photon: } \hbar\omega - \hbar\omega_0 \equiv \hbar\Delta = \frac{1}{2m} (p^2 - p_i^2) = \frac{1}{2m} (\cancel{\omega \hbar \vec{k} \cdot \vec{p}} + \cancel{\hbar^2 k^2})$$

$$\text{Doppler shift Recoil energy } E_r \equiv \frac{\hbar^2 k^2}{2m}$$

$\Rightarrow \Delta > 0$ for atoms moving away from light
 $\Delta < 0$ for atoms moving towards light.

Even for stationary atoms, we need to provide recoil energy.

$$\begin{aligned} \text{Atom: } \Delta E &= p_f^2/2m - p_i^2/2m = \frac{1}{2m} [\cancel{\omega p_i \cdot \hbar(k-k')} + \hbar^2 (k-k')^2] \\ &= \frac{1}{2m} [\cancel{\omega \vec{p}_i \cdot \hbar \vec{k}} - \cancel{\omega \vec{p}_i \cdot \hbar \vec{k}'} + \hbar^2 (k^2 + k'^2 - 2kk')] \end{aligned}$$

Assume k' is in random direction $\langle k' \rangle = 0$, and $|k| \approx |k'|$

$$\langle \Delta E \rangle = \frac{\hbar}{m} p_i \cdot k + \Delta \frac{\hbar^2 k^2}{2m}$$

Doppler cooling \propto recoil heating

Conclusion: photons push atoms \rightarrow Doppler effect

Doppler cooling occurs when $p_i \cdot k < 0$

\Rightarrow we need counter-propagating photons &
laser detuning $\Delta = \omega - \omega_0 < 0$.

Non-trivial conclusion: cooling limit is set by recoil energy = $1/\mu k$.

How do we include decay in quantum mechanics?

$$\begin{array}{c} \overline{\uparrow \downarrow} \\ \text{1e} \otimes \text{1p} \otimes \text{1n} \\ \text{1g} \otimes \text{1p} \otimes \text{1n} \end{array} \leftarrow \begin{array}{l} \text{photon field} \\ \uparrow \text{external state} \end{array}$$

7. Density matrix approach

$$\text{In rotating frame } i\hbar\partial_t \phi = \frac{1}{2} [\Omega \sigma_x - \Delta \sigma_z] \phi, \quad \phi = \begin{pmatrix} \phi_e \\ \phi_g \end{pmatrix}$$

$$\rho = |1\rangle\langle 1| \Rightarrow \rho_{ij} = \langle i|1\rangle\langle 1|j\rangle = \begin{cases} \rho_{22} = |\phi_e|^2 & \leftarrow \text{prob. in the ex state} \\ \rho_{11} = |\phi_g|^2 & \leftarrow \text{prob. in the ground state} \\ \rho_{12} = \phi_e^* \phi_g & \leftarrow \text{coherence} \end{cases}$$

$$\dot{\rho}_{22} = i \frac{\Omega}{2} (\rho_{21} - \rho_{12}) - T \rho_{22}$$

$$\rho_{11} + \rho_{22} = 1$$

$$\dot{\rho}_{11} = -i \frac{\Omega}{2} (\rho_{21} - \rho_{12}) + T \rho_{12}$$

$$\dot{\rho}_{22} = -\dot{\rho}_{11}$$

$$\dot{\rho}_{12} = -i \Delta \rho_{12} + i \frac{\Omega}{2} (\rho_{22} - \rho_{11}) - \frac{T}{2} \rho_{12}$$

Show that we get the optical Bloch eqn

↑
added by hand
to simulate decay.

$$i\vec{b} = (\Omega, 0, -\Delta) \times \vec{b} \\ - T/2 [bx, by, \Delta bz + z]$$

$$\text{Show that in equilibrium } \rho_{22} = \frac{-\Omega^2/4}{\Delta^2 + \Omega^2/4 + T^2/4}$$

Since intensity $I \propto E^2 \propto \Omega^2$, we define $\frac{I}{I_{\text{sat}}} = \frac{\Omega^2}{\Delta^2 + \Omega^2/4 + T^2/4}$

$$\Rightarrow \rho_{22} = \frac{1}{2} \frac{I/I_s}{1 + I/I_s + 4\Delta^2/\Omega^2}$$

saturation intensity

is the excited state population in equilibrium

\Rightarrow photon scattering rate $S = T \rho_{22} = \sigma I / h\nu$. σ : radiative cross section

$$\Rightarrow \sigma = \frac{\sigma_{\text{max}}}{1 + I/I_s + 4\Delta^2/\Omega^2}, \quad \sigma_{\text{max}} = \frac{3\lambda^4}{2\pi}$$

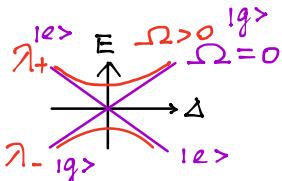
Radiative force and dipole force

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$$\text{Radiative force } f = \sigma \langle \vec{P}_f - \vec{P}_i \rangle = \Gamma \rho_{22} \hbar \langle \vec{k} - \vec{k} \rangle = \Gamma \rho_{22} \hbar \vec{k}$$

$$= \frac{\Gamma}{2} \frac{I/I_s}{1 + I/I_s + 4\Delta^2/\Gamma^2} \hbar k$$

$$\text{Dipole force } i\partial_t \phi = \frac{i}{2} \left[-\frac{\omega}{\Omega} \frac{\Omega}{\Delta} \right] \phi \quad \text{eigenenergy } \lambda \pm = \pm \sqrt{\Delta^2 + \Omega^2}$$



Consider we bring an atom close to a laser beam



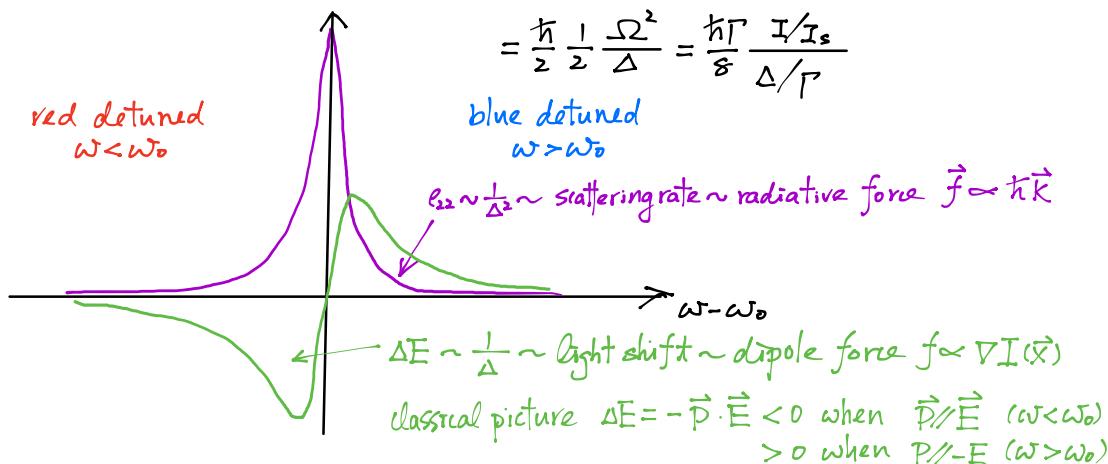
laser intensity

blue detuned $\omega > \omega_0$ excited atom feels attraction

$|g\rangle$ atom feels a repulsive potential

red detuned $\omega < \omega_0$ atom feels an attractive potential

Dipole potential for $|g\rangle$ is $V(x) = \frac{\hbar}{2} (\sqrt{\Delta^2 + \Omega^2} - \Delta)$



Quantum field treatment. (atom + photons)

$$H = H_{\text{atom}} + H_{\text{photons}} + H_{\text{a-p}}$$

$$H_{\text{atom}} = \hbar \frac{\omega_0}{2} \sigma_z$$

$$H_{\text{photons}} = \sum_i (b_i^\dagger b_i + \frac{1}{2}) \hbar \omega_i \equiv b^\dagger b \hbar \omega = n \hbar \omega$$

$$\begin{aligned} \text{Quantum state of the system } | > &= |\text{atom} > \otimes |\text{photon} > \\ &= |e > \otimes |n > \\ &= |e >, n > \end{aligned}$$

$$\begin{aligned} b |n > &= \sqrt{n} |n-1 > \\ b |0 > &= 0 \\ b^\dagger |n > &= \sqrt{n+1} |n > \\ \Rightarrow b^\dagger b |n > &= n |n > \end{aligned}$$

Absorption of photon

$$|g, n > \rightarrow |e, n-1 >$$

Stimulated emission of photon $|e, n > \rightarrow |g, n+1 >$
spontaneous emission $|e, 0 > \rightarrow |g, 1 >$

} Thus stimulated & spontaneous emission are related.

$$H_{\text{a-p}} = \frac{\hbar g}{2} (\sigma_+ b + \sigma_- b^\dagger)$$

$$\sigma_+ |g > = |e > \quad \sigma_+ |e > = 0$$

$$\sigma_\pm = \frac{1}{2} (\sigma_x \pm i \sigma_y)$$

$$\sigma_- |e > = |g > \quad \sigma_- |g > = 0$$

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$H = \frac{\hbar \omega_0}{2} \sigma_z + \hbar \omega b^\dagger b + \frac{\hbar g}{2} (\sigma_+ b + \sigma_- b^\dagger) \quad \Psi = \begin{matrix} \downarrow \text{state of atom} \\ |e >, n > \\ \nwarrow \text{state of photon, } n=0, 1, 2, \dots \end{matrix}$$

σ_z and $b^\dagger b$ are diagonal in the basis of $|e >, n >$

Starting with $|g, n >$, the interaction term couples $|g, n >$ to $|e, n-1 >$

$$(\sigma_+ b + \sigma_- b^\dagger) |g, n > = \sqrt{n} |e, n-1 >$$

$$(\sigma_+ b + \sigma_- b^\dagger) |e, n-1 > = \sqrt{n} |g, n >$$

Quantum states are pairwisely coupled.

We may build a subspace $\Psi = \alpha |g, n > + \beta |e, n-1 > \equiv \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$

$$H = \frac{1}{2} \hbar \omega_0 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \hbar \omega \begin{bmatrix} n-1 & 0 \\ 0 & n \end{bmatrix} + \frac{\hbar g}{2} \begin{pmatrix} 0 & \sqrt{n} \\ \sqrt{n} & 0 \end{pmatrix}$$

$$= \frac{\hbar}{2} (\omega_0 - \omega) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \hbar \omega \begin{bmatrix} n-\frac{1}{2} & 0 \\ 0 & n-\frac{1}{2} \end{bmatrix} + \frac{\hbar g}{2} \sqrt{n} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Compare with rotating frame $H = -\frac{\Delta}{2} \sigma_z + \frac{\Omega}{2} \sigma_x$

They are essentially the same if we call $\Omega = g \sqrt{n}$

$$\frac{1}{\hbar} H = -\frac{\Delta}{2} \sigma_z + \frac{g \sqrt{n}}{2} \sigma_x + \cancel{\text{const.}} \Rightarrow \lambda_\pm = \frac{1}{2} \sqrt{\Delta^2 + ng^2}$$

