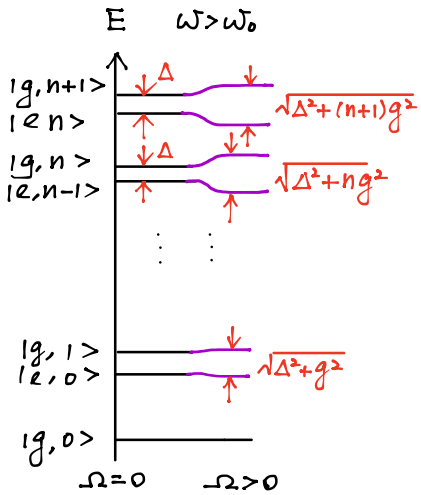
Cavity QED, dressed atom picture, Jayne-Cummings model 10/21/2021



Energy splitting can be compared to generalized Rabi freq $\Omega_R = \sqrt{\Delta^2+\Omega^2}$

$\Rightarrow$ Rabi freq $\Omega = \frac{1}{\hbar}\langle e|\, d\cdot E\,|g\rangle \equiv \frac{\tilde{d}E}{\hbar}$

$= \sqrt{n}\, g$

How is n related to E?

Total energy $= n\hbar\omega =$ energy density $\cdot V$

$= (\epsilon_0 E^2/2)V$

$\Rightarrow n = \frac{\epsilon_0 E^2 V}{2\hbar\omega} \Rightarrow \hbar g = \frac{\tilde{d}\,\cancel{E}}{\cancel{E}}\sqrt{\frac{2\hbar\omega}{\epsilon_0 V}} = \tilde{d}\,\mathcal{E}$

$|e.0\rangle$ couples to $|g.1\rangle$ with matrix element $\langle g|V|e\rangle = \frac{1}{2}\hbar g = \frac{1}{2}\tilde{d}\,\mathcal{E}$ ← vacuum fluctuation

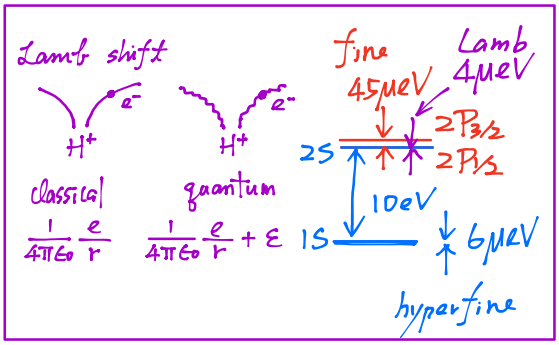
In reality $|e.0\rangle$ couples to all modes with the same frequency ω

Fermi's golden rule gives $\Gamma_{ge} = \frac{2\pi}{\hbar}|\langle g|V|e\rangle|^2 \rho(E)$ ← density of state $\rho(E) = \frac{V\omega^2}{\hbar\pi^2 c^3}$

$= \frac{2\pi}{\hbar}\frac{1}{4}\tilde{d}^2\frac{2\hbar\omega}{\epsilon_0 \cancel{V}}\frac{\cancel{V}\omega^2}{\hbar\pi^2 c^3}$

$= \frac{\langle d^2\rangle\omega^3}{3\pi\epsilon_0\hbar c^3}$ note ω^3 dependence $d \nearrow \longrightarrow E$ $\tilde{d}^2 = \langle d^2\rangle/3$

$\mathcal{E} = \sqrt{2\hbar\omega/\epsilon_0 V}$ is the field of the ground state of the photon field. It leads to spontaneous emission and couples to all modes equally.



Einstein A, B coefficients.

spontaneous emission

stimulated absorption
stimulated emission

$\dot{N}_e = -A N_e$ $\quad \dot{N}_e = B_{eg} N_g \mathcal{E} - B_{ge} N_e \mathcal{E}$

$= -\dot{N}_g$ $\quad = -\dot{N}_g$

$\Rightarrow$ from the above calculation we easily conclude $B_{eg} = B_{ge}$

$A = B\, 8\pi h\nu^3/c^3$,

which Einstein derived from thermodynamics.