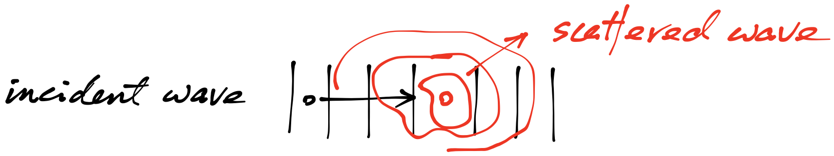
Lecture 12 Atomic scattering at low temperatures 11/9/2021

What determines the coupling constant g in the GP eqn?

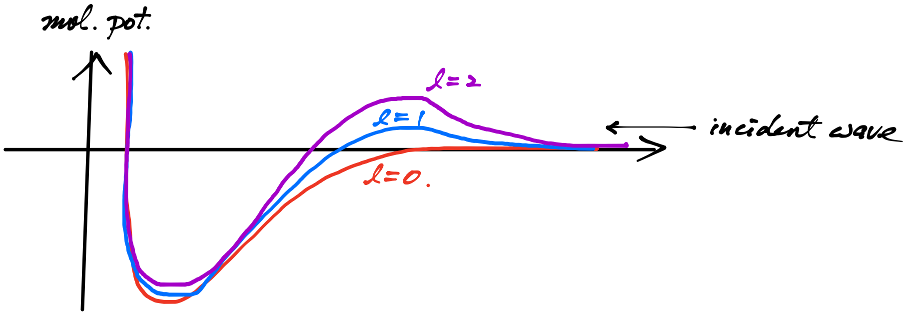
Consider a box $V(x)$. when we introduce the 1st particle



$$\varphi = \varphi_{inc} + \varphi_s = e^{ikz} + f(\theta,\phi)\frac{e^{ikr}}{r}$$

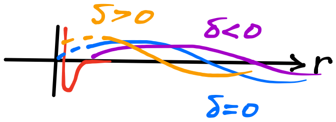
$$= e^{ikz} + \sum_{lm} f_{lm} Y_{lm}(\theta,\phi)\frac{e^{ikr}}{r}$$

At low-temp. only head on collisions happen (s-wave $l=m=0$)



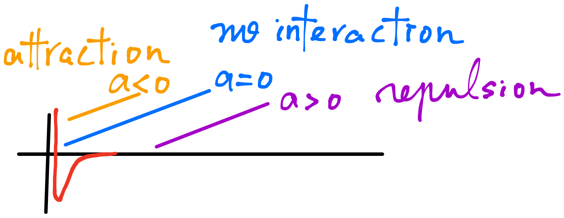
scattering matrix $S \equiv e^{i2\delta}$ ← scattering phase shift

$$\varphi = \underbrace{e^{-ikr}}_{\text{incident wave}} - \overset{\downarrow}{S}\,\overbrace{\frac{e^{ikr}}{r}}^{\text{outgoing wave}}$$



$k \to 0$ $\varphi = 1 - \frac{a}{r}$

In the low energy limit



Scattering length: offset of the $l=0$ radial wavefunction when $k \to 0$

From 2-body to many-body interaction.

model the interaction as a pseudo potential $V(r) \equiv g\,\delta(r)$

s.t. $\varphi \propto 1-\frac{a}{r}$ at low energy $k \to 0$

$$-\frac{\hbar^2}{2\mu}\nabla^2\varphi = g\,\delta(r)\,\varphi$$

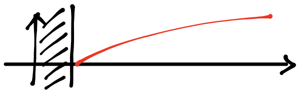
$$\Rightarrow g\,\delta(r)\,\varphi = -\frac{\hbar^2}{m}\nabla^2\varphi = \frac{-\hbar^2}{m}\nabla^2\left(1-\frac{a}{r}\right) = \frac{4\pi\hbar^2 a}{m}\,\delta(r)$$

$$\Rightarrow g = \frac{4\pi a\hbar^2}{m}\,\delta(r)$$

$\Rightarrow$ Gross-Pitaevskii Eqn $\left[\frac{\hbar^2 k^2}{2m} + V + \frac{4\pi a\hbar^2}{m}|\varphi|^2\right]\varphi = \mu\varphi$

Example: square barrier: $V(r<A)=\infty$

$V(r>A)=0$



square well $V(r<A)=-V_0$

$V(r>A)=0$