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Final Solution

Cheng Chiu

1. 2. All we need at $t=0$ is the b.c. for the homogeneous solution
2. 2. Three particles should have 3 modes in 1D, but 1 is translation.
3. 2.3. General solutions are $F(x-vt)$, $G(x+vt)$ & their superpositions.
4. no answer $\vec{\nabla} A^2 = 2i \partial_i A_j A_j = 2i A_j \partial_i A_j = 2(\vec{A} \cdot \partial_x \vec{A}, \vec{A} \cdot \partial_y \vec{A}, \vec{A} \cdot \partial_z \vec{A})$
5. 4. even function and mean = $\pi/2$
6. 1.4. $d \sin \theta = n \lambda \Rightarrow$ finite # of fringes, $d \uparrow \theta \downarrow, \lambda \uparrow \theta \uparrow$
7. 2.3.4. molecules can always move faster due to Boltzmann distribution
Snell's law, Huygen's principle & Fermat's principle apply to all waves.
Since nobody goes faster than sound speed. No boom.

$$2. a \quad x_1'' + \frac{\omega_1}{Q_1} x_1' + \omega_1^2 x_1 = \omega_1^2 W$$

$$\Rightarrow \gamma_1 = \frac{\omega_1}{Q_1} = \frac{2\pi}{T_1 Q_1}, \quad k_1 = m_1 \omega_1^2 = m_1 \frac{4\pi^2}{T_1^2}, \quad f_1 = \omega_1^2 W$$

$$\text{given } W = e^{i\omega t}, \quad x_1 = A e^{i\omega t} \Rightarrow [(\omega_1^2 - \omega^2) + \gamma_1 i \omega] A = \omega_1^2 W$$

$$\Rightarrow A = \left| \frac{\omega_1^2}{\omega_1^2 - \omega^2 + i\gamma_1 \omega} \right| = \frac{\omega_1^2}{\sqrt{(\omega_1^2 - \omega^2)^2 + \gamma_1^2 \omega^2}}$$

$$\text{Worse case } A = \frac{\omega_1}{\gamma_1} = Q_1$$

$$b. \quad x_1'' + \gamma_1 x_1' + \omega_1^2 x_1 = \omega_1^2 W + \frac{m_2}{m_1} \omega_2^2 (x_2 - x_1)$$

$$x_2'' + \gamma_2 x_2' + \omega_2^2 x_2 = \omega_2^2 x_1$$

$$W = e^{i\omega t} \Rightarrow x_1 = A_1 e^{i\omega t}$$

$$x_2 = A_2 e^{i\omega t}$$

$$\Rightarrow [(\omega_1^2 - \omega^2) + i\omega \gamma_1] A_1 = \omega_1^2 + \frac{m_2}{m_1} \omega_2^2 A_2$$

$$[(\omega_2^2 - \omega^2) + i\omega \gamma_2] A_2 = \omega_2^2 A_1$$

$$\Rightarrow A_2 = \left| \frac{\omega_2^2}{(\omega_2^2 - \omega^2) + i\omega \gamma_2} \right| A_1$$

$$c. \quad x_1'' + \gamma_1 x_1' + \omega_1^2 x_1 = \omega_1^2 e^{i\omega t} + \frac{m_2}{m_1} \omega_2^2 x_2$$

$$x_2'' + \gamma_2 x_2' + \omega_2^2 x_2 = \omega_2^2 x_1$$

$$\vec{X} = \vec{A} e^{i\omega t} \Rightarrow \gamma_1 i \omega_1 A_1 = \omega_1^2 + \frac{m_2}{m_1} \omega_2^2 A_2$$

$$[(\omega_2^2 - \omega_1^2) + i\gamma_2 \omega_1] A_2 = \omega_2^2 A_1$$

$$\Rightarrow \left[i\gamma_1 \omega_1 - \frac{m_2}{m_1} \omega_2^2 \frac{\omega_2^2}{\omega_2^2 - \omega_1^2 + i\gamma_2 \omega_1} \right] A_1 = \omega_1^2$$

$$\text{Set } \omega_2 = \omega_1 \text{ \& } \gamma_2 = 0 \Rightarrow A_1 = 0.$$

$$\Rightarrow \omega_1^2 + \frac{m_2}{m_1} \omega_1^2 A_2 = 0 \Rightarrow A_2 = -\frac{m_1}{m_2} \Rightarrow \text{amp. } \frac{m_1}{m_2}$$

d. In general $w = e^{i\omega t} \Rightarrow x_1 = A_1 e^{i\omega t} \quad x_2 = A_2 e^{i\omega t}$

$$\Rightarrow [(\omega_1^2 - \omega^2) + i\gamma_1 \omega_1] A_1 = \omega_1^2 + \frac{m_2}{m_1} \omega_2^2 A_2$$

$$[(\omega_2^2 - \omega^2) + i\gamma_2 \omega_2] A_2 = \omega_2^2 A_1 \quad \text{set } \omega_2 = \omega_1$$

$$\Rightarrow [(\omega_1^2 - \omega^2) + i\gamma_1 \omega_1 - \frac{m_2}{m_1} \omega_1^4 \frac{1}{\omega_1^2 - \omega^2 + i\gamma_2 \omega_1}] A_1 = \omega_1^2$$

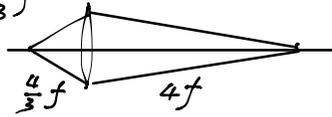
$$\Rightarrow [(\omega_1^2 - \omega^2) + i\gamma_1 \omega_1 - \frac{m_2}{m_1} \omega_1^4 \frac{\omega_1^2 - \omega^2 - i\gamma_2 \omega_1}{\sqrt{(\omega_1^2 - \omega^2)^2 + \gamma_2^2 \omega_1^2}}] A_1 = \omega_1^2$$

$\omega_1 = \omega$ imaginary part $\gamma_1 \omega_1 + \frac{m_2}{m_1} \omega_1^4 \frac{1}{\gamma_2 \omega_1}$

$$= (\gamma_1 + \frac{m_2}{m_1} \frac{\omega_1^3}{\gamma_2}) \omega_1 \equiv \gamma_{\text{eff}} \omega_1 \Rightarrow \gamma_{\text{eff}} \approx \gamma_1 + \frac{m_2}{m_1} \frac{\omega_1^2}{\gamma_2}$$

3. a) $\frac{1}{x} + \frac{1}{D_i} = \frac{1}{f} \Rightarrow D_i = \frac{xf}{x-f} > 0 \Rightarrow x > f$

$x = \frac{4}{3}f$



$$\Rightarrow M = -\frac{D_i}{D_o} = \frac{f}{f-x}$$

b). $D_o' = d - D_i \quad \frac{1}{D_o'} + \frac{1}{D_i'} = \frac{1}{f'} \Rightarrow D_i' = \frac{D_o' f'}{D_o' - f'}$

$$\Rightarrow MM' = \frac{f}{f-x} \frac{f'}{f' - D_o'} = \frac{f}{f-x} \frac{f'}{f' - d + \frac{xf}{x-f}} = \frac{ff'}{(f-x)(f'-d) - xf}$$

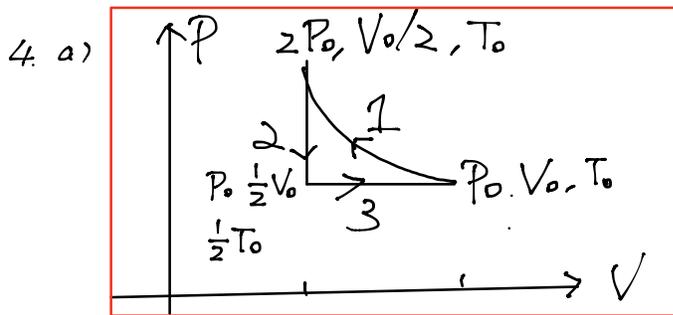
c) $d \rightarrow 0 \Rightarrow MM' = \frac{f}{f-x - xf/f'} = \frac{f}{f - (1+f/f')x} \Rightarrow f_{\text{eff}} = \frac{f}{1+f/f'} = \frac{ff'}{f+f'}$

d) For small x , $|MM'|$ gets larger when x increases.

MM' flips sign when $(f-x^*)(f'-d) = x^*f$

$$\Rightarrow x^*(f+f'-d) = f(f'-d) \Rightarrow x^* = \frac{f(f'-d)}{f+f'-d}$$

Given x^* , $f' = d + \frac{fx^*}{f-x^*}$



b). Step 1 $\frac{3}{2} P_0 V_0$ P_0 V_0 T_0

Step 2 $\frac{3}{2} P_0 V_0$ $2P_0$ $V_0/2$ T_0

Step 3 $\frac{3}{4} P_0 V_0$ P_0 $V_0/2$ $T_0/2$

c) Step 1 $\Delta W = \int P \Delta V = \int_{V_0}^{V_0/2} \frac{NKT_0}{V} dV = -P_0 V_0 \ln 2$

$$\Delta Q = \Delta U + P \Delta V = -P_0 V_0 \ln 2$$

$$T_0 \Delta S = \Delta Q \Rightarrow \Delta S = -\frac{P_0 V_0 \ln 2}{T_0}$$

Step 2 $\Delta W = 0$ $\Delta Q = \Delta U = -\frac{3}{4} P_0 V_0$

$$\Delta S = \int \frac{dQ}{T} = \int \frac{dU}{T} = \frac{3}{2} NK \int_{T_0}^{T_0/2} \frac{dT}{T} = -\frac{3}{2} NK \ln 2 = -\frac{3}{2} \ln 2 \frac{P_0 V_0}{T_0}$$

Step 3 $\Delta W = \int P \Delta V = \frac{1}{2} P_0 V_0$, $\Delta Q = \Delta U + P \Delta V = \frac{3}{4} P_0 V_0 + \frac{1}{2} P_0 V_0 = \frac{5}{4} P_0 V_0$

$$\Delta S = \int \frac{dQ}{T} = \int \frac{dU}{T} + \frac{P dV}{T} = \frac{3}{2} NK \int_{T_0/2}^{T_0} \frac{dT}{T} + NK \int \frac{dV}{V}$$

$$= \frac{3}{2} NK \ln 2 + NK \ln 2 = \frac{5}{2} \ln 2 \frac{P_0 V_0}{T_0}$$

d). Total work done: $(\frac{1}{2} - \ln 2) P_0 V_0 < 0$

Total heat absorbed: $(-\ln 2 - \frac{3}{4} + \frac{5}{4}) P_0 V_0 = (\frac{1}{2} - \ln 2) P_0 V_0 < 0$

Total change of entropy: $(-\ln 2 - \frac{3}{2} \ln 2 + \frac{5}{2} \ln 2) \frac{P_0 V_0}{T_0} = 0$

e). The system absorbs external work, and delivers heat \Rightarrow refrigerator