HW6 solution Cheng Chin

1a. $T \propto \sqrt{x^2+a^2} + \sqrt{(d-x)^2+b^2}\,(n_2/n_1)$

$$\partial_x T = 0 \Rightarrow \frac{x}{\sqrt{x^2+a^2}} - \frac{d-x}{\sqrt{(d-x)^2+b^2}}\frac{n_2}{n_1} = 0 \Rightarrow n_1 \sin\alpha = n_2 \sin\beta$$

b. Light would not know all possible paths to follow one with the shortest path. It does take any path with transverse time at local minimum.

2a. $\nabla\times(\nabla\times E) = -\partial_t \nabla\times B = -\mu\epsilon\partial_t^2 E - \mu\sigma\partial_t E = \nabla(\nabla\cdot E) - \nabla^2 E$

$$\Rightarrow \mu\epsilon\partial_t^2 E + \mu\sigma\partial_t E = \nabla^2 E - \frac{1}{\epsilon}\nabla\rho$$

$$\nabla\times(\nabla\times B) = \mu\epsilon\partial_t \nabla\times E + \mu\nabla\times j = -\mu\epsilon\partial_t^2 B + \mu\sigma\nabla\times E = \nabla(\nabla\cdot B) - \nabla^2 B$$

$$\Rightarrow \mu\epsilon\partial_t^2 B + \mu\sigma\partial_t B = \nabla^2 B$$

b. $\partial_t^2 E + \frac{\sigma}{\epsilon}\partial_t E = \frac{1}{\mu\epsilon}\nabla^2 E$. Ansatz $E = e^{i(kz-\omega t)} \Rightarrow k^2 = \mu\epsilon\omega^2 + i\omega\mu\sigma$

Assume $k = k + i/z_0 \Rightarrow k^2 = k^2 - z_0^{-2} + 2ik/z_0$

$\Rightarrow k/z_0 = \omega\mu\sigma/2$, $k^2 - z_0^{-2} = \omega^2\mu\epsilon$

$\Rightarrow z_0 = 2k/\omega\mu\sigma \Rightarrow k^4 - \omega^2\mu\epsilon k^2 - \omega^2\sigma^2\mu^2/4 = 0$

$\Rightarrow k^2 = \frac{1}{2}(\omega^2\mu\epsilon \pm \sqrt{\omega^4\mu^2\epsilon^2 + \omega^2\sigma^2\mu^2})$ choose real root.

$\Rightarrow v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}\frac{1}{2}(1+\sqrt{1+\sigma^2/\omega^2\epsilon^2})^{-1/2}$

$z_0 = 2/v_p\mu\sigma$.

c. $\sigma \gg \mu/\epsilon$. Approximate the above result:

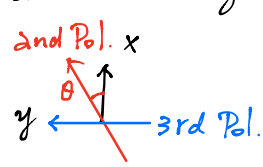
$$v_p = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\frac{2\omega\epsilon}{\sigma}} \Rightarrow \omega = k\frac{1}{\sqrt{\mu\epsilon}}\sqrt{\frac{2\omega\epsilon}{\sigma}} = k\sqrt{\frac{2\omega}{\mu\sigma}}$$

$$\Rightarrow \omega = k^2\, 2/\mu\sigma, \quad d\omega/dk = 2\sqrt{2\omega/\mu\sigma} = 2v_p$$

$$v_p \approx c\sqrt{\frac{2\omega\epsilon}{\sigma}} = 0.07c \qquad z_0 = 2\times10^{-9} = 2\,\text{nm}$$

3. a 2nd polarizer would not allow E_x to transmit $\Rightarrow$ No light goes through.

b After 1st one $\Rightarrow \vec{E} = E_x(1,0)$



After 2nd one $\Rightarrow$ $\vec{E} = E_x \cos\theta\,(\cos\theta, \sin\theta)$

After 3rd one $\Rightarrow$ $\vec{E} = E_x \cos\theta\,(0, \sin\theta) = E_x\,(0, \sin\theta\cos\theta)$

c. Above shows that for $E_{in} = (E_x, 0)$, $E_{out} = E_x \cos\theta\,(\cos\theta, \sin\theta)$

If incident beam is $E_{in} = (0, E_y)$, similar calculation gives

$$E_{out} = E_y \sin\theta\,(\cos\theta, \sin\theta)$$

Thus from superposition principle, $E_{in} = (E_x, E_y)$ gives $E_{out} =$

$$\begin{pmatrix} E_x \cos^2\theta + E_y \sin\theta\cos\theta \\ E_x \cos\theta\sin\theta + E_y \sin^2\theta \end{pmatrix} = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$