

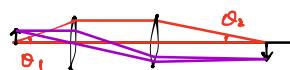
HW7 Solution Cheng Chin

1 a) Lens eqn: $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}$

1st lens: $\frac{1}{x} + \frac{1}{D_i} = \frac{1}{f_1}$. when $x = f_1$, $D_i = \infty \Rightarrow$ outgoing beam is collimated.

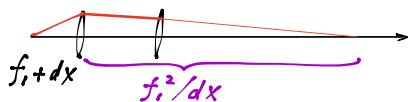
2nd lens: $\frac{1}{\infty} + \frac{1}{y} = \frac{1}{f_2} \Rightarrow y = f_2 \Rightarrow$ CCD should be placed f_2 away from tube lens.

b)



The ratio of the sizes is $f_2/f_1 = D_1/\theta_2 = \text{magnification}$

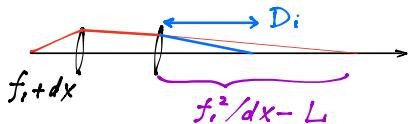
c) 1st lens $\frac{1}{f_1+dx} + \frac{1}{D_i} = \frac{1}{f_1} \Rightarrow \frac{1}{f_1}(1 - \frac{dx}{f_1}) + \frac{1}{D_i} = \frac{1}{f_1} \Rightarrow D_i = \frac{f_1^2}{dx}$



2nd lens $-\frac{1}{f_1^2/dx + L} + \frac{1}{D_i} = \frac{1}{f_2} \Rightarrow \frac{1}{D_i} = \frac{1}{f_2} + \frac{dx}{f_1^2 - L dx} \approx \frac{1}{f_2} + \frac{dx}{f_1^2}$

$$\Rightarrow D_i = \frac{f_2 f_1^2 / dx}{f_2 + f_1^2 / dx} \approx f_2(1 - f_2 dx / f_1^2) = f_2 - (\frac{f_2}{f_1})^2 dx$$

$$\Rightarrow dy = -(\frac{f_2}{f_1})^2 dx$$



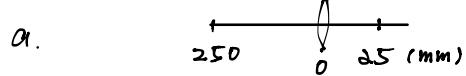
d) 1st lens magnification $M_1 = -\frac{f_1^2 / dx}{f_1 + dx}$

2nd lens magnification $M_2 = -\frac{f_2 - (\frac{f_2}{f_1})^2 dx}{f_1^2 / dx - L}$

$$\begin{aligned} \text{Total magnification } M_1 M_2 &= \frac{f_1^2 / dx}{f_1 + dx} \frac{f_2 - (\frac{f_2}{f_1})^2 dx}{f_1^2 / dx - L} = \frac{f_2(1 - f_2 dx / f_1^2)}{(f_1 + dx)(1 - \frac{L}{f_1^2} dx)} \\ &= \frac{f_2}{f_1} \frac{1 - f_2 dx / f_1^2}{1 + dx / f_1 - L / f_1^2 dx} \approx M_0 + M_0 \left(-\frac{f_2}{f_1^2} + \frac{L}{f_1^2} - \frac{1}{f_1} \right) dx \end{aligned}$$

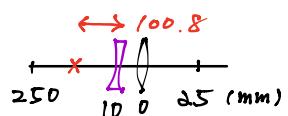
$$\Rightarrow dM = \frac{f_2}{f_1} \frac{L - f_1 - f_2}{f_1^2} dx = \frac{f_2(L - f_1 - f_2)}{f_1^3} dx \quad \text{when } L = f_1 + f_2 \quad dM/dx = 0.$$

2.



$$\frac{1}{250} + \frac{1}{25} = \frac{1}{f} \Rightarrow f = 22.73 \text{ mm}$$

b.



$$\frac{1}{250} + \frac{1}{D_i} = -\frac{100.8}{1000} \Rightarrow D_i = -100.8 \text{ mm}$$

2nd lens $\frac{1}{100.8} + \frac{1}{25} = \frac{1}{f} \Rightarrow f = 20.39 \text{ mm}$

$$\frac{1}{D_i} + \frac{1}{25} = \frac{1}{20.39} \Rightarrow \text{Effective position of book } D_i = 111 \text{ mm}$$

c. $22.73 - 20.39 = 2.34 \text{ mm}$

3a. $r' = [(x-x')^2 + L^2]^{1/2} = L [1 + \left(\frac{x-x'}{L}\right)^2]^{1/2} = L + \frac{(x-x')^2}{2L}$

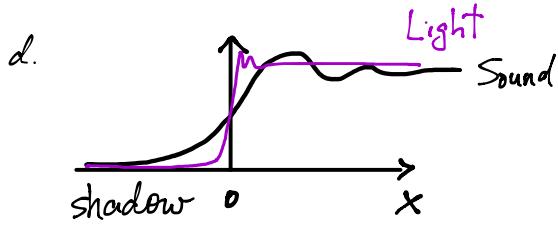
$$\begin{aligned} E(x, L) &= \int E_{in}(x') \frac{e^{i(kL - \omega t)}}{L} e^{i k (x-x')^2 / 2L} dx' \\ &= \frac{1}{L} e^{i(kL - \omega t)} \int E_{in}(x') e^{i k (x-x')^2 / 2L} dx' \\ E_{in} &= E_0 \delta(x-d/2) + E_0 \delta(x+d/2) \\ E(x, L) &= \frac{E_0}{L} e^{i(kL - \omega t)} (e^{i k (d/2 - x)^2 / 2L} + e^{i (d/2 + x)^2 / 2L}) \\ &= \frac{E_0}{L} e^{i(kL - \omega t)} e^{i[k(d/2)^2 + x^2] / 2L} (e^{i k x d / 2L} + e^{-i k x d / 2L}) \\ &= \pm \frac{E_0}{L} e^{i(kL - \omega t)} e^{i[k(d/2)^2 + x^2] / 2L} \cos k x d / 2L \end{aligned}$$

$\Rightarrow \text{Maxima: } k x d / 2L = 0, \pm \pi, \pm 2\pi, \dots$

$$\Rightarrow d x / L = \frac{2}{\lambda} n \neq n\pi = n\lambda, \quad n=0, \pm 1, \dots$$

$$\begin{aligned} b. E(x, L) &= \frac{1}{L} e^{i(kL - \omega t)} E_0 \int_{-d/2}^{d/2} e^{i k (x-x')^2 / 2L} dx' \\ &= \frac{1}{L} e^{i(kL - \omega t)} E_0 e^{i k x^2 / 2L} \int_{-d/2}^{d/2} e^{-i k x' x / L} e^{i k x'^2 / 2L} dx' \\ &\approx \frac{E_0}{L} e^{i(kL - \omega t)} e^{i k x^2 / 2L} \underbrace{\int_{-d/2}^{d/2} e^{-i k x' x / L} dx'}_{\sin k d x / 2L} = \frac{2L}{kx} \sin k d x / 2L \\ &= \frac{E_0 d}{L} e^{i(kL - \omega t)} e^{i k x^2 / 2L} \frac{\sin k d x / 2L}{k d x / 2L}, \text{ identical to textbook 9.32g (35)} \end{aligned}$$

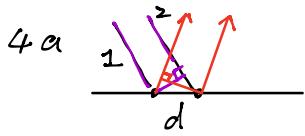
$$\begin{aligned}
 c. E(x, L) &= \frac{1}{L} e^{i(kL - \omega t)} E_0 \int_0^\infty e^{ik(x+x')^2/L} dx' \\
 &= \frac{\sqrt{2L}}{k} \int_{-\sqrt{kSL}/x}^\infty e^{iu^2} du \quad u = \sqrt{k(x-x')^2/L} \\
 &= \frac{\sqrt{2L}}{k} \int_{-\sqrt{kSL}/x}^0 e^{iu^2} du + \int_0^\infty e^{iu^2} du \quad \text{Fresnel function} \\
 &= \frac{\sqrt{2L}}{k} \left[\int_0^{\sqrt{kSL}/x} e^{iu^2} du + \sqrt{\frac{\pi}{8}}(1+i) \right] \quad C(x) = \int_0^x \cos u^2 du \\
 &= \frac{\sqrt{2L}}{k} \left[C(\sqrt{kSL}/x) + i S(\sqrt{kSL}/x) + \sqrt{\frac{\pi}{8}}(1+i) \right] \quad S(x) = \int_0^x \sin u^2 du \\
 &\text{take real part} \quad = \frac{\sqrt{2L}}{k} [C(\sqrt{kSL}/x) + \sqrt{\frac{\pi}{8}}]
 \end{aligned}$$



~~Both light and sound can go around the corner and reach the screen 4 away within the length of $\sqrt{2L}/k = \sqrt{L\lambda/\pi}$~~

Assume $L = 10\text{m}$. sound wavelength $\approx 2\text{kHz}$ is $30\text{cm} \Rightarrow \sqrt{L\lambda/\pi} = 1\text{m}$
 light wavelength (green) is $500\text{nm} \Rightarrow \sqrt{L\lambda/\pi} = 1\text{mm}$

\Rightarrow light only extends 1mm into the shadow, sound extends 1m.



1st beam comes in with incident angle i , and is diffracted with refracted angle θ_n

$$\text{Beam path difference} = L_2 - L_1 = d \sin i - d \sin \theta_n = n\lambda$$

b. $-d \sin \theta_1 = \lambda$

$$-d \cos \theta_1, \Delta \theta_1 = \Delta \lambda$$

$$\Rightarrow \Delta \theta_1 / \theta_1 = \Delta \lambda / \lambda \tan \theta_1 = \Delta \lambda / \lambda \frac{-\lambda/d}{\sqrt{1-\lambda^2/d^2}} = -\frac{\Delta \lambda}{d} (1-\lambda^2/d^2)^{-1/2}$$

$$\text{For } \lambda = 589 \text{ nm}, d = \frac{1}{1200} \text{ mm} \Rightarrow \sin \theta_1 = -\lambda/d = 0.707 \Rightarrow \theta_1 = -45^\circ$$

$$\Rightarrow \tan \theta_1 = 1 \Rightarrow \Delta \theta_1 / \theta_1 = \Delta \lambda / \lambda \Rightarrow \Delta \theta_1 = 45^\circ \frac{0.6}{589} = 0.046^\circ$$

c. $2d \sin \theta_1 = \lambda \Rightarrow \theta_1 = \sin^{-1} \frac{\lambda}{2d} = 50^\circ$

When grating rotates by $d\phi$, incident angle changes by $di = d\theta_1 - d\phi$

$$|\Delta \lambda / d\phi| = d\lambda / d\theta_1 = 2d \cos \theta_1 = 2d \cos 50^\circ = 714 \text{ nm/rad} = 0.714 \text{ nm/mrad}$$