Physics 143b: Honors Waves, Optics, and Heat

Spring Quarter 2024

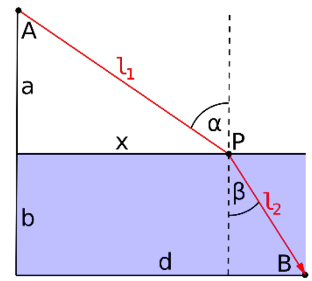
Problem Set #7

Due: 11:59 pm, Thursday, May 8. Please submit to Canvas.

1. Fermat's principle (10 points each)

   We can derive Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$ from Maxwell's equations. However, Maxwell's equations were completed in 1861, Snell's law was discovered a few centuries earlier by many, including Snellius (1580~1626) and P. Fermat in 1662. Here we will see how Fermat derived the law based on the *Fermat principle* of least time:

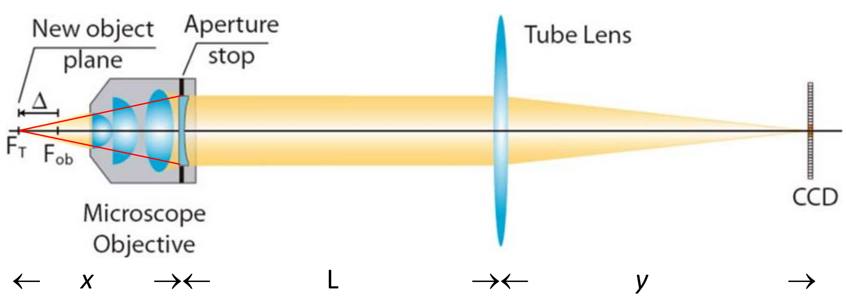
   > ***"The path taken by a ray between two given points is the path that can be traversed in the*** ***least time****."*

   Given the index of refraction of air *n=n*$_1$ and water *n=n*$_2$, a ray passes point A above the water and point B below the water. See figure for the dimensions. Show that when the travel time $T(x) = \frac{l_1(x)}{c/n_1} + \frac{l_2(x)}{c/n_2}$ is minimized, you get Snell's law $n_1 \sin\alpha = n_2 \sin\beta$.

2. Optical microscope. (7.5 points each)

   A generic design of an optical microscope is illustrated below

   We consider an object on the left side of the microscope objective with the working distance of *x*, the distance between the object and the objective.) For simplicity, we assume the objective is a thin lens, and it collimates the light coming from the object onto the tube lens. The tube lens then forms an image on the CCD. Assume the objective has a focal length of f_1 and the tube lens has a focal length of f_2 .

   a) Use the lens equation $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}$ and show that when $x = f_1$, the CCD should be placed away from the tube lens by exactly $y = f_2$ to form a clear image. The distance between the two lenses *L* does not matter. (This is called infinite conjugation). Draw the ray diagram for light coming from the object and show that rays do converge to the CCD.

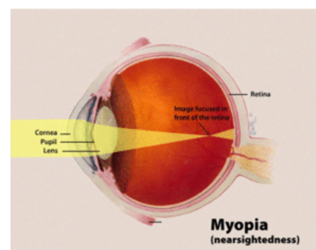
(Hint: For right-propagating rays, use the convention that D_0 is defined to be positive if the object is on the left of the lens and $D_i > 0$ if the image on the right of the lens.)

b) Under the condition $x = f_1$ and $y = f_2$ Draw the ray diagram for an object slightly off the image axis and show that the image on the CCD is upside down and is magnified by a factor of $M_0 = f_2/f_1$.
(Hint: Magnification of a single lens is given by $M = -D_i/D_0$. For compound lenses, it is the product of all magnifications.)

c) When the working distance deviates from the above condition slightly $x = f_1 + dx$, where $dx \ll f_1, f_2, L$. (This frequently happens in real experiments), one would need to move the position of CCD to $y = f_2 + dy$ in order to get a clear image again? Determine *dy* in terms *dx.*

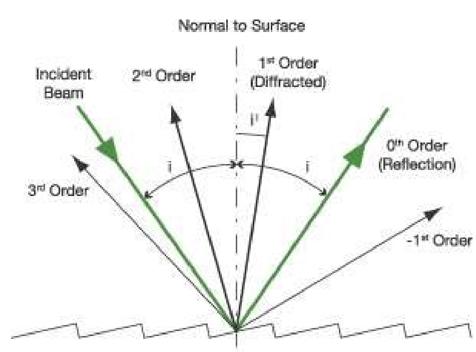
d) Following c), when the image is reoptimized, how much is the magnification modified $M = M_0 + dM$? Determine *dM* in terms of *dx* and show such correction to the magnification vanishes when $L = f_1 + f_2$.

3. Myopia correction (10 points each)
Myopia occurs when the incident light from a point source is focused by the lens in front of the retina, see figure, while a healthy eye focuses it on the retina.
The distance between the lens and the retina is effectively 25mm (it is effective so we may consider the index of refraction $n \approx n_{air} \approx 1$ behind the lens of the eyeball.).



a) What would be the focal length of the lens of a healthy person when he/she is reading a book 25 cm away from the eyes.
b) John's reading eyeglasses has a refractive power of -5.75 diopters (D), which is defined as the reciprocal of the focal length in meter, namely, $f = \frac{1}{-5.75} m$, and it is placed 10mm in front of his eyeballs. Show that it effectively brings the book 11.08 cm in front of his eyes.
c) Show that the focal length of his eyeball lens is 2.33 mm shorter than that of a healthy person.

4. Diffraction grating (10 points each)
Diffraction grating is the critical component behind a large number of optical devices, including DVD, spectrometers, digital displays and so on. Grating is a reflective surface with periodic structure, see figure. Assuming the periodicity of the structure is *d* and a beam with wavelength λ illuminates the grating with an incident angle i. Show that in addition to the regular reflection, we have diffracted beams with different orders propagating in different directions.



a) The *n*-th order refracted beam originates from the constructive interferences of scattered beams with reflection angle θ_n, where $n = 0, \pm 1, \pm 2$ ... In particular, the regular reflected beam can be considered as the 0th-order refraction with $\theta_0 = i$. Show that

$$d(\sin i - \sin \theta_n) = n\lambda,$$

b) German scientist Joseph von Fraunhofer first noted the sodium absorption in the solar spectrum. The Sodium absorption contains two very closely spaced lines near $\lambda \approx$589.0 and 589.6nm, which can be distinguished from their slightly different diffraction angle. Consider the sunlight goes straight down to the grating (incident angle $i = 0$) with periodicity *d*=(1/1200) mm, calculate the angles of the 1st order diffraction θ_1 and show that for small wavelength difference $\Delta\lambda$, we have

$$\frac{\Delta\theta}{\theta} = -\frac{\Delta\lambda}{d}\left(1 - \frac{\lambda^2}{d^2}\right)^{-1/2}$$

c) Diffraction gratings are also used in quantum optics experiments to fine tune the laser wavelength at the precision of 10 fm (10^{-14}m) by optical feedback. Let's consider a laser beam is oriented toward the grating and feedback occurs when the first order diffraction goes exactly back toward the laser source $\theta_1 = -\theta_i$. How would you set the incident angle θ_i of a laser beam toward a grating with 1800 lines per meter to stabilize a laser at the wavelength λ =852 nm. Show that you can fine tune the laser wavelength $d\lambda$ by rotating the diffraction grating angle by $d\phi$. Determine the tuning sensitivity $d\lambda/d\phi$ of the laser.