Midterm

1 A Yes underdamped $\rightarrow \gamma < 2\omega_0$. So higher k will still give $\gamma < 2\omega_0$.

B No either both underdamped. critical damped or overdamped.

C False when driven. response is oscillatory.

D. indep. V'' does not change

E. $\omega < \omega_0$ Max amp occurs @ $\omega = \sqrt{\omega_0^2 - \gamma^2/2} < \omega_0$

F. $2\omega_0$ Force const is doubled, mass halved $\Rightarrow \omega = \sqrt{2k/(m/2)} = 2\omega_0$

G. longer longer string gives low ω_0. easier to satisfy $\gamma > 2\omega_0$

H. CH_3 more particles more normal modes.

2. (a)
$$m_1 x_1'' = -k_1(x_1 - x_2)$$
$$m_2 x_2'' = k_1(x_1 - x_2) - k_2(x_2 - x_3)$$
$$m_3 x_3'' = k_2(x_2 - x_3)$$
$$\Rightarrow \hat{M} = \begin{pmatrix} -k_1/m_1 & k_1/m_1 & 0 \\ k_1/m_2 & -(k_1+k_2)/m_2 & k_2/m_2 \\ 0 & k_2/m_3 & -k_2/m_3 \end{pmatrix}$$

$$\hat{M} = -\omega_0^2 \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

(b) $$\hat{M}\vec{A} = -\omega^2 \vec{A} \Rightarrow \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} \vec{A} = \frac{\omega^2}{\omega_0^2} \vec{A} \equiv \lambda \vec{A}$$

$$\left| \begin{pmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{pmatrix} \right| = (\lambda-1)^2(2-\lambda) + (\lambda-1) + (\lambda-1) = 0$$

$$\Rightarrow (\lambda-1)[(\lambda-1)(2-\lambda)+2] = (\lambda-1)(-\lambda^2+3\lambda) = 0 \Rightarrow \lambda(\lambda-1)(\lambda-3) = 0$$

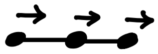
$$\Rightarrow \omega = 0,\ \pm\omega_0,\ \pm\sqrt{3}\omega_0$$

For $\lambda = 0$, $\omega = 0$, $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} \vec{A} = 0 \Rightarrow \vec{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

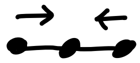
For $\lambda = 1$. $\omega = \pm\omega_0$ $\begin{pmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{pmatrix} \vec{A} = 0 \Rightarrow \vec{A} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

For $\lambda = 3$. $\omega = \pm\sqrt{3}\omega_0$ $\begin{pmatrix} -2 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix} \vec{A} = 0 \Rightarrow \vec{A} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

(c) mode 1: $\vec{X} = (X_0 + v_0 t)\begin{pmatrix}1\\1\\1\end{pmatrix}$

center of mass translation 

mode 2: $\vec{X} = C\cos(\omega_0 t + \phi)\begin{pmatrix}1\\0\\-1\end{pmatrix}$

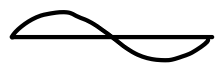
Sym. vibration 

mode 3: $\vec{X} = C\cos(\sqrt{3}\omega_0 t + \phi)\begin{pmatrix}1\\-2\\1\end{pmatrix}$

Asym. vibration 

3. A. Since the function is anti-sym. $f(x) = -f(-x)$ all sym terms $C_n = 0$.

B. $\int_0^{2\pi} \sin mx\, f(x) = \int_0^{2\pi} \sin^2 mx\, d_m\, dx = d_m \pi$

$= \int_0^{\pi} \sin mx\, dx - \int_{\pi}^{2\pi} \sin mx\, dx$ 

$= \frac{1}{m}\int_0^{m\pi} \sin u\, du - \frac{1}{m}\int_{m\pi}^{2m\pi} \sin u\, du = 0$ when m is even

$= \frac{4}{m}$ when m is odd

$\Rightarrow d_m = \frac{4}{\pi m}$ for $m = 1.3.5\cdots$ $\quad d_m = 0$ for $m = 2.4.6\cdots$

$\Rightarrow f(x) = \frac{4}{\pi}\sum_{m=1.3.5\cdots} \frac{1}{m}\sin mx$

c. $f(k) = \frac{1}{2\pi}\int e^{-ikx} f(x)\, dx$

$= \frac{1}{2\pi}\frac{4}{\pi}\sum_{m=1.3\cdots} \int e^{-ikx}\frac{1}{2i}\left(e^{imx} - e^{-imx}\right) dx$

$= \frac{1}{\pi^2 i}\sum_{m=1.3\cdots} \int e^{i(m-k)x} - e^{i(-m-k)x}\, dx$

$= \frac{1}{\pi^2 i}\sum_{m=1.3\cdots} \left[\delta(k-m) + \delta(k+m)\right] = \frac{1}{\pi^2 i}\sum_{m=\pm1,\pm3\cdots} \delta(k-m)$