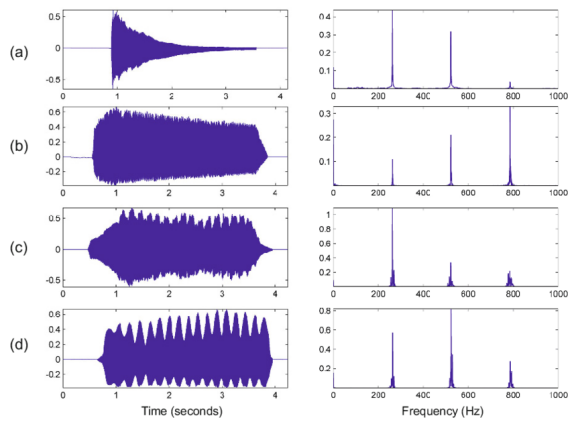
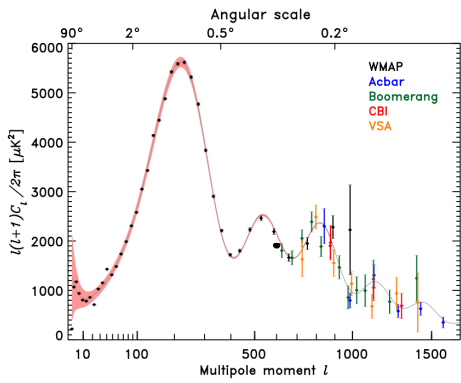
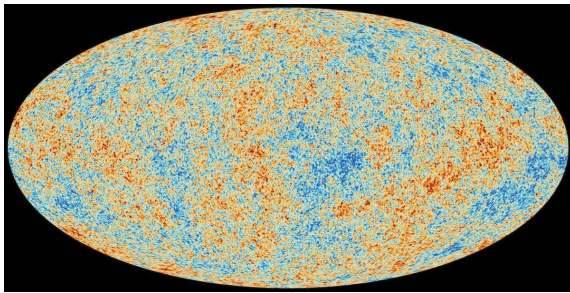


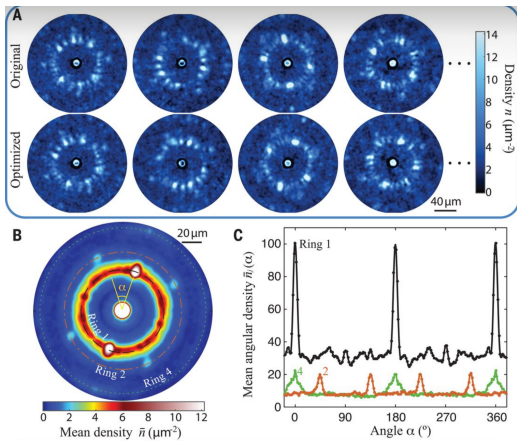
Examples of Fourier spectra



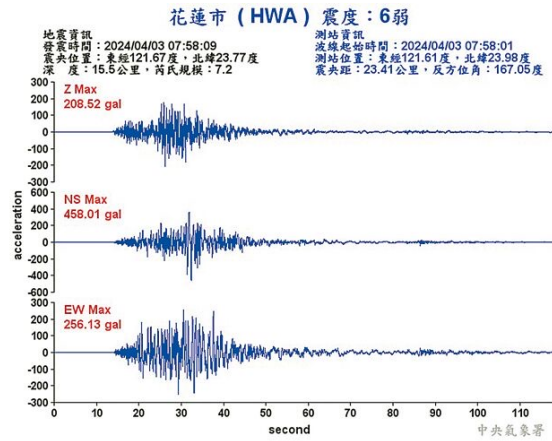
Trumpet, piano, violin, flute



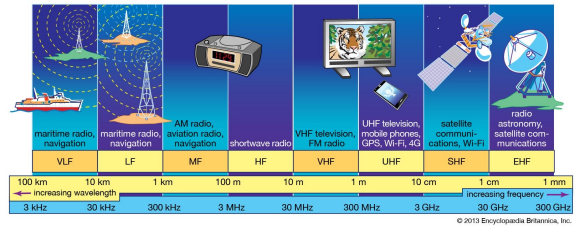
cosmic microwave background.



spontaneous formation of patterns in quantum gas.



seismic waves

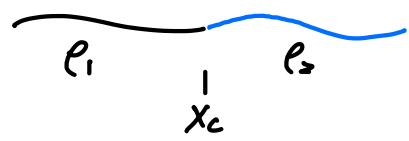


telecommunication spectrum



Optical fiber noise

Connecting two strings of diff. density ρ_1, ρ_2 $v_1 = \frac{\omega_1}{k_1}$ $v_2 = \frac{\omega_2}{k_2}$



Waves going to diff medium
Water, light, seismic and others.

What do we do @ the boundary?

General consideration: $\omega_1 = \omega_2$ and $T_1 = T_2 \Rightarrow k_1 : k_2 = \sqrt{\rho_1} : \sqrt{\rho_2} = v_2 : v_1$

assume $x_c = 0$.

incident
 \downarrow
 $\psi(x < 0) = A e^{i(k_1 x - \omega t)} + B e^{i(-k_1 x - \omega t)}$
reflection
 \downarrow

$\psi(x > 0) = \psi_T = C e^{i(k_2 x - \omega t)}$
transmission
 \leftarrow

Continuity

$\psi_L(x_c^-, t) = \psi_R(x_c^+, t)$

No force at the interface

$\psi'_L(x_c^-, t) = \psi'_R(x_c^+, t)$

$\psi(0^-) = \psi(0^+) \Rightarrow A + B = C$
 $\psi'(0^-) = \psi'(0^+) \Rightarrow i k_1 (A - B) = i k_2 C$

$\Rightarrow \begin{cases} A + B = C \\ A - B = \frac{k_2}{k_1} C \end{cases} \Rightarrow$

reflection: $\frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2} = \frac{\sqrt{\rho_1} - \sqrt{\rho_2}}{\sqrt{\rho_1} + \sqrt{\rho_2}} = \frac{v_2 - v_1}{v_1 + v_2}$
 transmission: $\frac{C}{A} = \frac{2k_1}{k_1 + k_2} = \frac{2\sqrt{\rho_1}}{\sqrt{\rho_1} + \sqrt{\rho_2}} = \frac{2v_2}{v_1 + v_2}$

check simple limits:

1. $\rho_1 = \rho_2$: Reflection $B \rightarrow 0$ $C \rightarrow 1$ ✓
2. $\rho_2 \gg \rho_1$ (hitting a wall) $C \rightarrow 0$, $B = -1$ ✓
3. $\rho_1 \gg \rho_2$ (hitting nothing) $C \rightarrow 2$, $B = 1$ ✓