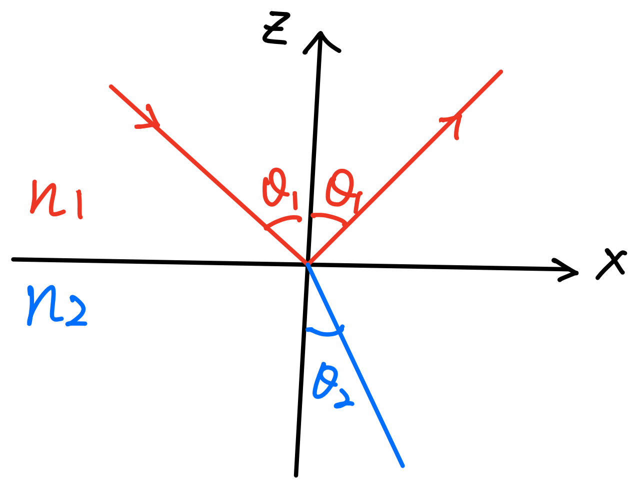
Geometrical Optics

Interface

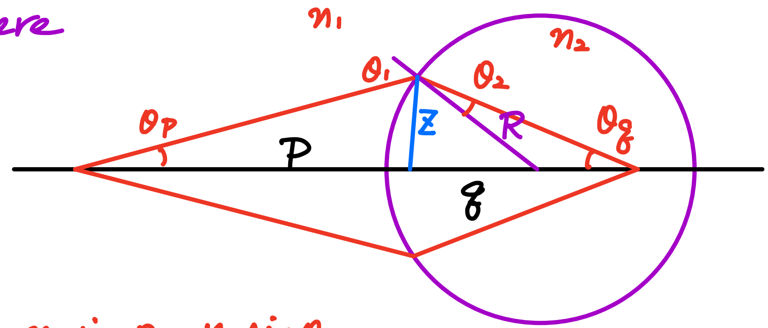
Snell's law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

see h.w.



Sphere



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\tan\theta_p = \frac{z}{p}$

$\tan\theta_q = \frac{z}{q}$

$\sin(\theta_2 + \theta_q) = \frac{z}{R}$

$\theta_1 = \theta_2 + \theta_q + \theta_p$

$\Rightarrow$ $\theta_2 = z/R - z/q$

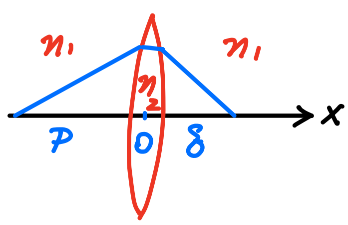
$\theta_1 = z/R + z/p$

$\sin\theta \simeq \tan\theta \simeq \theta$

$\Rightarrow n_1(1/R + 1/p) = n_2(1/R - 1/q)$

$\Rightarrow \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

lens : combination of 2 interfaces from n_1 to n_2 to n_1



1st interface : $\frac{n_1}{p} + \frac{n_2}{Q} = \frac{n_2 - n_1}{R_1}$

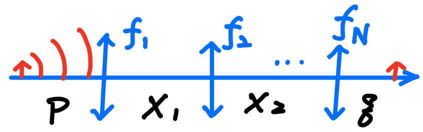
2nd interface : $\frac{n_2}{-Q} + \frac{n_1}{q} = \frac{n_1 - n_2}{-R_2}$

source is on the right side

Add 2 Eqns $\Rightarrow$ $\boxed{\frac{1}{p} + \frac{1}{q} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \equiv \frac{1}{f}}$ ← focal length

thin lens formula

Lens system:



@ 1st lens: $\frac{1}{p}+\frac{1}{q_1}=\frac{1}{f_1}$

@ 2nd lens: $\frac{1}{x_1-q_1}+\frac{1}{q_2}=\frac{1}{f_2}$

⋮

@ Nth lens: $\frac{1}{x_{N-1}-q_{N-1}}+\frac{1}{q}=\frac{1}{f_N}$

p is positive if the source is on the incident side

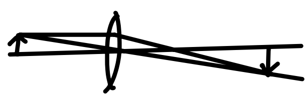
q is positive if the light converges on the outgoing side.

Applications:

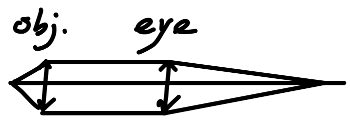
1. Collimation & focusing

$\frac{1}{p}+0=\frac{1}{f}$ $\qquad$ $0+\frac{1}{q}=\frac{1}{f}$

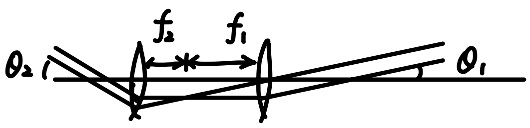
2. Magnifier / μscope



$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ Magnification $=\frac{q}{p}$

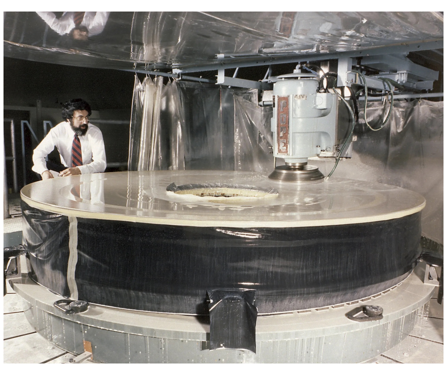


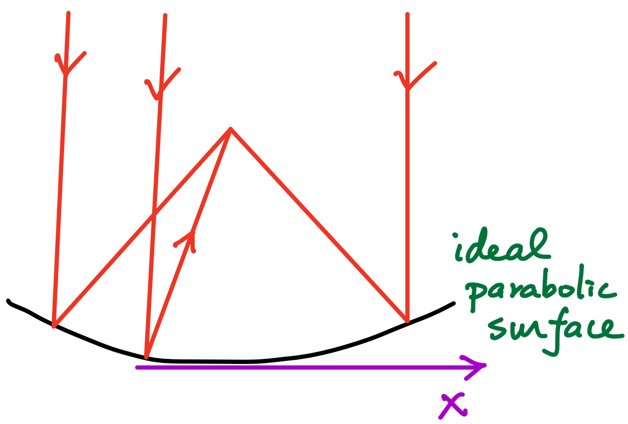
3. telescope: angular magnification



Angular mag. $= f_1/f_2$

4. reflecting telescope



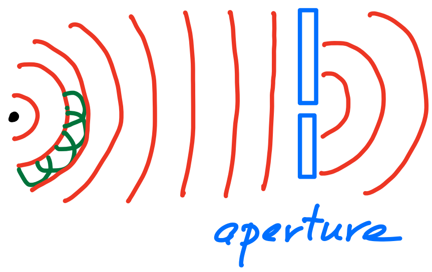


Hubble's mirror flaw: 1.3 mm too flat on the 2.4 m mirror

Diffraction

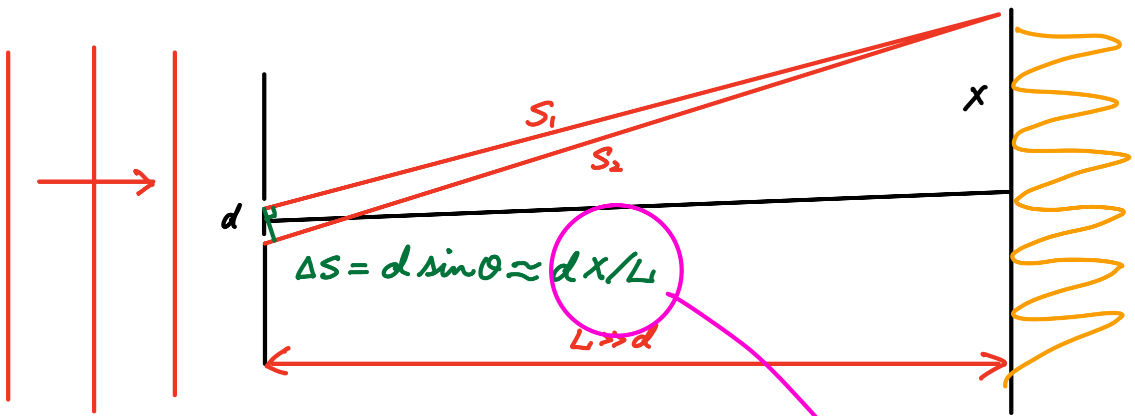
Huygens-Fresnel principle

Every point on a wavefront is itself a source of spherical wave



In spherical coordinate, point source generate $\psi(r,\theta,\phi) = \psi_0 \frac{e^{i(kr-\omega t)}}{r}$ in the far field

Young's double-slit exp



Intensity on the screen $I = c\epsilon|E|^2$

$$E = E_1 + E_2 = \frac{1}{S_1}e^{i(kS_1-\omega t)} + \frac{1}{S_2}e^{i(kS_2-\omega t)}$$

$$\approx \frac{1}{\langle S\rangle}e^{-i\omega t}\underbrace{\left[e^{ikS_1}+e^{ikS_2}\right]}$$

$S_1 = \sqrt{L^2+(x-d/2)^2}$

$S_2 = \sqrt{L^2+(x+d/2)^2}$

constructive interference: kS_1 and kS_2 are in phase $|e^{i\phi}+e^{i\phi}|^2 = 4$

destructive interference: kS_1 and kS_2 are out of phase $|e^{i\phi}-e^{i\phi}|^2 = 0$

$$I = c\epsilon\frac{1}{L^2}\left|1+e^{ik(S_2-S_1)}\right|^2 = 4I_0\cos^2\tfrac{1}{2}\Delta\phi$$

I_0: intensity of a single beam

$\Delta\phi = \phi_2 - \phi_1 = k(S_2 - S_1)$

max when $\Delta\phi = 0, \pm 2\pi, \pm 4\pi \cdots \Rightarrow \Delta S = 0, \pm\lambda, \pm 2\lambda \cdots$

min when $\Delta\phi = \pm\pi, \pm 3\pi \cdots \Rightarrow \Delta S = \pm\lambda/2, \pm 3\lambda/2 \cdots$

Consider max: $S_1 - S_2 = N\lambda$, $N = 0, \pm 1, \pm 2 \cdots$

First of all. $|S_1 - S_2| < d$, at most $1 + d/\lambda$ maxima.

If $d < \lambda$ only 1 max at the center.

Consider $\lambda \ll d \ll L$

$$|S_1 - S_2| = \sqrt{L^2 + (x + d/2)^2} - \sqrt{L^2 + (x - d/2)^2} = N\lambda$$

$$= L\left[1 + \frac{1}{2}\left(\frac{x + d/2}{L}\right)^2\right] - L^2\left[1 + \frac{1}{2}\left(\frac{x - d/2}{L}\right)^2\right]$$

$$= L \frac{1}{2} \frac{1}{L^2} 4 x d/2 = \frac{dx}{L}$$

$\Rightarrow$ location of the n-th max is $x_n = \frac{L}{d} n\lambda$ Wavelength amplified by $L/d = 10^3 \sim 10^4$!!

Intensity $= 4 I_0 \cos^2 \frac{k}{2}(S_2 - S_1) = 4 I_0 \cos^2 \frac{1}{2}\frac{d}{L} k x$

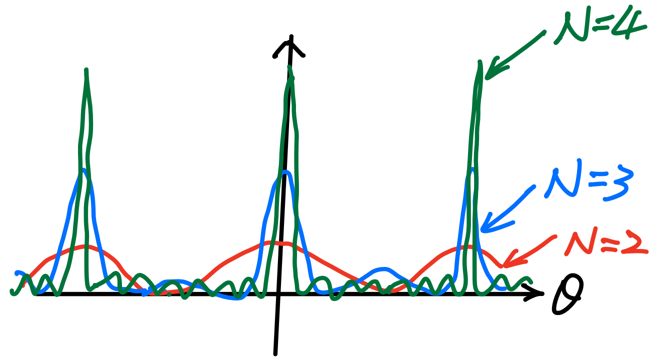
Generalized to N-slit.

$\alpha = ikd\sin\theta$

$$E = E_0 + E_1 + \cdots + E_{N-1} = E\sum_{j=0}^{N-1} e^{ikS_j} = E\sum e^{ikjd\sin\theta} \equiv E\sum_{j=0}^{N-1} e^{\alpha j}$$

$$= E\,\frac{\alpha^N - 1}{\alpha - 1} \qquad 1 + 2 + 4 + 8 = 16 - 1$$

$$\Rightarrow I = I_0 \frac{\sin^2 N\pi d\sin\theta/\lambda}{\sin^2 \pi d\sin\theta/\lambda}$$

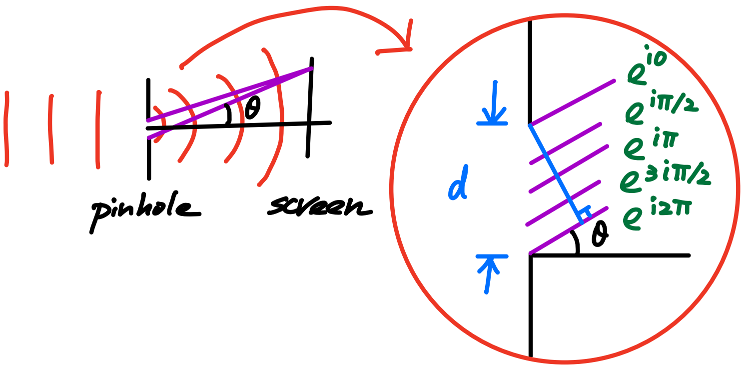


Peak value: $I = N^2 I_0$ N slits give N^2 higher intensity at max.

width $N\pi d\sin\theta/\lambda = \pi \Rightarrow \Delta\theta = \frac{\lambda}{Nd}\pi$ decreases with N.

Thus total energy $\int I(\theta)\,d\theta = N^2 I_0 \frac{\lambda\pi}{Nd} \propto N$ still proportional to # of slits.

Single slit diffraction

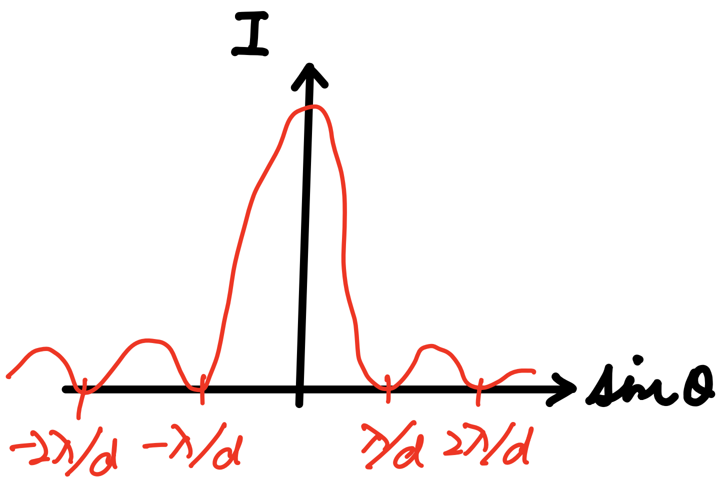


max: $\theta = 0$, $d\sin\theta = \pm\lambda/2, \pm 3\lambda/2, \cdots$

min: $d\sin\theta = \pm\lambda, \pm 2\lambda \cdots$

When $d\sin\theta = N\lambda$, a beam with phase $e^{i\theta}$ is always paired with one with $e^{i\theta + i\pi} = -e^{i\theta}$

$\Rightarrow$ Complete destructive interference.



$$E = \int_{-d/2}^{d/2} A e^{ikS(x)} dx = A e^{ikS_0} \int_{-d/2}^{d/2} e^{-i\frac{2\pi}{\lambda} x \sin\theta} dx = A e^{ikS_0} \frac{2}{k\sin\theta} \sin\frac{kd\sin\theta}{2}$$

$$|E|^2 = 4A \frac{\sin^2 \frac{kd\sin\theta}{2}}{k^2 \sin^2\theta}$$

numerator goes to 0 when $d\sin\theta = 0, \pm\lambda, \pm 2\lambda \cdots$

denominator goes to 0 when $\sin\theta = 0$.

In the limit of $\theta \to 0$ $|E|^2 = 4A \lim_{\theta \to 0} \frac{k^2 d^2 \theta^2 / 4}{k^2 \theta^2} = Ad^2$ is actually the max