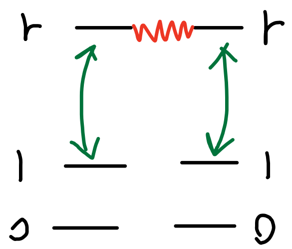
Realization of CNOT gate

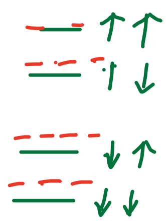
tweezer array: Rydberg blockade



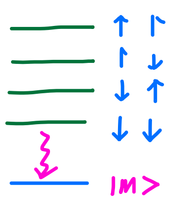
Strong int. when both atoms are in the Rydberg state.

Optical lattice: phase gate

spin dependent interaction.

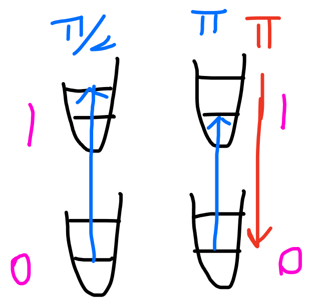


molecules



phase gate.

Ion trap: Cirac-Zoller gate.



$|0.0;\nu=0\rangle$

$\to |1.0.\nu=1\rangle + |0.0.0\rangle$

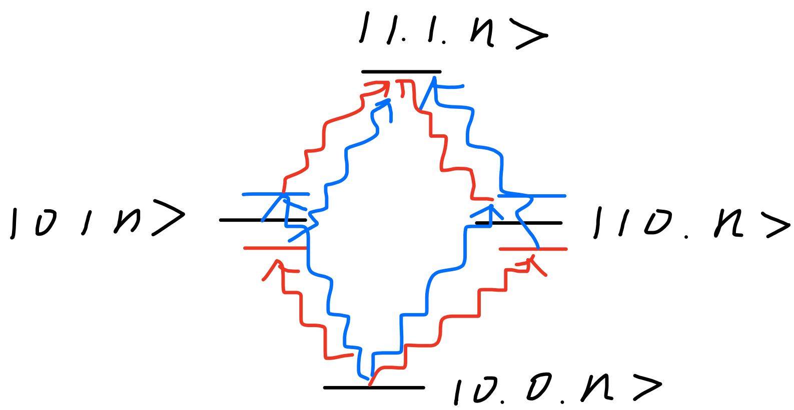
$\to |1.1.\nu=1\rangle + |0.1.0\rangle$

$\boxed{\to |1.0;0\rangle + |0.1.0\rangle}$

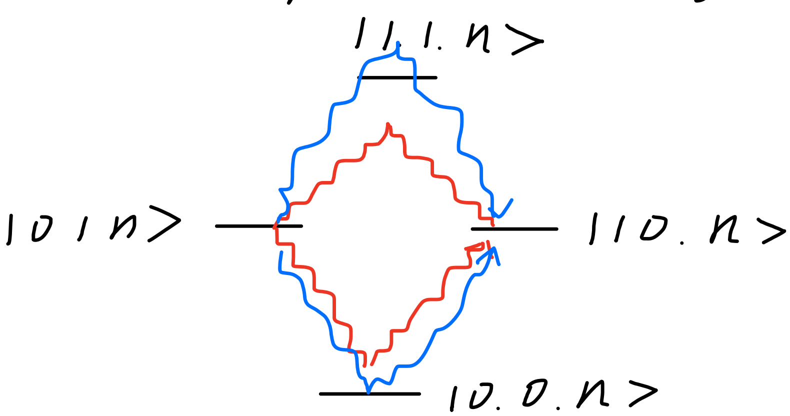
Key step. where excitation goes from 1st to 2nd ion.

Require ions in the ground state.

Molmer-Sorensen gate



$|0.0\rangle$ couples $|1.1\rangle$ through 2-photon



$|0.1\rangle$ couples to $|1.0\rangle$ through 2 photons.

Overall transition

$$
U = \begin{array}{c c}
 & \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\
\begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} &
\begin{pmatrix}
\cos\Omega t/2 & 0 & 0 & \sin\Omega t/2 \\
0 & \cos\Omega t/2 & \sin\Omega t/2 & 0 \\
0 & -\sin\Omega t/2 & \cos\Omega t/2 & 0 \\
-\sin\Omega t/2 & 0 & 0 & \cos\Omega t/2
\end{pmatrix}
\end{array}
$$

Second quantization

$$|\ \rangle = \sum_i a_i |i\rangle$$

$$\varphi(x) = \int \phi(k) e^{-ikx} dk$$
$$= \sum \varphi_k e^{-ikx}$$

2 atoms $\varphi_B(x_1, x_2) = \varphi_B(x_1, x_2)$

$$\varphi_F(x_1, x_2) = -\varphi_F(x_1, x_2)$$

$$\varphi_B(x_1, x_2) = \sum \overset{A_{12}}{\overset{\parallel}{\varphi_{k_1}\varphi_{k_2}}} e^{-ik_1x_1} e^{-ik_2x_2}$$
$$\varphi_B(x_2, x_1) = \sum \varphi_{k_1}\varphi_{k_2} e^{-ik_1x_2} e^{-ik_2x_1}$$
$$= \sum \underbrace{\varphi_{k_2}\varphi_{k_1}}_{=A_{21}} e^{-ik_1x_1} e^{-ik_2x_2}$$

$$\Rightarrow \varphi_{k_1}\varphi_{k_2} = \varphi_{k_2}\varphi_{k_1} \Rightarrow A_{ij} = A_{ji}$$

If each particle can reach N eigenstates
2 indep particles have N^2 eigenstates.
2. bosonic particles $N(N+1)/2$ states
2 fermionic particles $N(N-1)/2$ states.

When $N=2$ (spin $1/2$). Classical : 4 states
Bosons : 3 states
Fermions : 1 states.

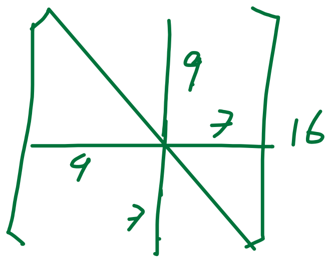
Example:

Cs has $F=3$, $F=4$ $\Rightarrow$ 16 ground states

Cs-Cs has $16\times17/2 = 136$ states.

If both in $F=3$.

$7\times8/2 = 28$ states



$F = 0, 2, 4, 6.$

states $= 1+5+9+13$

both in $F=4$

$9\times10/2 = 45$ states

$F = 0, 2, 4, 6, 8$

states $= 1+5+9+13+17$

$= 45$

one in $F=3$, one in $F=4$ $\Rightarrow$ 63 states

$F = 1, 2, 3 \cdots 7$

states $= 3+5+7+\cdots 15 = 63.$

2nd quantization

$|\,\rangle = |n_1, n_2 \cdots n_N\rangle$ $\qquad |0010001\rangle$

n_i = # of atoms in the ith eigenstate

$\sum n_i$ = total # $\qquad |0200000\rangle$

N = number of single particle states

Degrees of freedom $= N(N-1)/2 + N = N(N+1)/2$

$$|1.1.000\rangle = (\varphi_1 \otimes \varphi_2 + \varphi_2 \otimes \varphi_1)/\sqrt{2}$$

$$|1.1.1.00\rangle = (\varphi_1\varphi_2\varphi_3 + \varphi_2\varphi_1\varphi_3 + \varphi_1\varphi_3\varphi_2 + \varphi_3\varphi_1\varphi_2 + \varphi_3\varphi_2\varphi_1 + \varphi_2\varphi_3\varphi_1)/\sqrt{6}$$

$$|N\text{particle}\rangle = (\sum \varphi_1 \cdots \varphi_N)/\sqrt{N!}$$ wrong

$$b_1^+|1.1\rangle = b_1^+(\varphi_1\varphi_2 + \varphi_2\varphi_1)/\sqrt{2}$$

3 terms

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}}(\varphi_1\varphi_1\varphi_2 + \cdots \varphi_1\varphi_2\varphi_1) + \frac{1}{\sqrt{3}}(\varphi_1\varphi_2\varphi_1 + \cdots \varphi_2\varphi_1\varphi_1)\right)$$

$$= \sqrt{2}\,\frac{1}{\sqrt{3}}(\varphi_1\varphi_1\varphi_2 \cdots + \varphi_2\varphi_1\varphi_1)$$

$$= \sqrt{2}\,|2.1\rangle$$

$$\frac{1}{\sqrt{3}}(\varphi_1\varphi_1\varphi_2 + \varphi_2\varphi_1\varphi_1 + \varphi_1\varphi_2\varphi_1)$$

$$= \frac{1}{\sqrt{12}}\sum \varphi_\alpha\varphi_\beta\varphi_\gamma$$

all permutations

$$|n_1, n_2 . n_3 \cdots\rangle = \frac{1}{\sqrt{N!\,n_1!\,n_2!\cdots}}\sum \varphi_\alpha\varphi_\beta\cdots$$

the state the 1st particle is permuted into.

Say 2 atoms in μ

1 atom in ν $\quad \psi = \frac{1}{\sqrt{3}}(\varphi_\mu\varphi_\mu\varphi_\nu + \varphi_\mu\varphi_\nu\varphi_\mu + \varphi_\nu\varphi_\mu\varphi_\mu)$

$|0.2.0\cdots\rangle \equiv |2\rangle \otimes |2\rangle$

$|1.2,0\cdots\rangle = \frac{1}{\sqrt{3}}(|1\rangle|2\rangle|2\rangle + |2\rangle|1\rangle|2\rangle + |2\rangle|2\rangle|1\rangle)$

$|1.1.1\cdots\rangle = \frac{1}{\sqrt{6}}(|1\rangle|2\rangle|3\rangle + \cdots |3\rangle|2\rangle|1\rangle)$

Fock space $\mathcal{H}^F = \mathcal{H}^0 \oplus \mathcal{H}^1 \oplus \mathcal{H}^2 \oplus \mathcal{H}^3 \oplus \cdots$

vacuum

direct sum

basis of Fock space $= |n_1, n_2 . n_3 \cdots\rangle$

Creation operator $a_i^+ |n_1 . n_2 \cdots\rangle$

$= \sqrt{n_i+1}\, |n_1 \cdots n_i+1 \cdots\rangle$

annihilation operator $a_i |n_1 \cdots n_i \cdots\rangle$

$= \sqrt{n_i}\, |n_1 \cdots n_i - 1 \cdots\rangle$

$\Rightarrow [a_i, a_j^+] = a_i a_j^+ - a_j^+ a_i = \delta_{ij}$ (Bosons)

$\{c_i . c_j^+\} = c_i c_j^+ - c_j^+ c_i = \delta_{ij}$ (Fermions)