Gross-Pitaevskii Equation

$$H = \sum_i (p_i^2/2m + V(r_i)) + \sum_{i<j} U(r_i - r_j)$$

Short-range potential $U(r) = 0$ for $r > 0$.

$$= \sum p_i^2/2m + V(r_i) + U_0 \sum_{i<j} \delta(r_i - r_j)$$

$$U_0 = 4\pi\hbar^2 a/m$$

$$\psi(x_1, x_2 \cdots x_N) = S[\phi_1(x_1)\phi_2(x_2)\cdots]$$

All atoms in the ground state $\phi_i = \phi$

$$\psi = \prod_{i=1}^{N} \phi(x_i)$$

$$\psi(x) \equiv \sqrt{N}\,\phi(x) \Rightarrow n(x) = |\psi(x)|^2$$

$$\int n(x) = N$$

Energy functional

$$\langle H \rangle = N \int \frac{\hbar^2}{2m} |\nabla\phi|^2 + V|\phi|^2 + \frac{1}{2} N U_0 |\phi|^4 \, dx$$

subject to $\int |\phi|^2 dx = 1$

$$E \text{ (Free energy)} = H - \mu N \int |\phi|^2 dx$$

$$\delta E = N\int d^3x \left[\frac{\hbar^2}{2m}\nabla\phi\nabla\delta\phi^* + V\phi\delta\phi^* + NU_0 |\phi|^2\phi\delta\phi^* - \mu\phi\delta\phi^*\right) + c.c.$$

$$\Rightarrow \left(-\frac{\hbar^2}{2m}\nabla^2 + V + U_0|\phi|^2\right)\phi = \mu\phi.$$

$$\varphi \equiv \sqrt{N}\,\phi$$