1. **Bose condensation**
   - **Bose statistics**
   - **Phase space density**
2. **Mean-field approach**
   - **Gross-Pitaevskii equation**
3. **Experiments**
   - **Time of flight**
   - **Ground state population**
   - **Expanding gas**
4. **Other interesting experiments**

Phase space density

Quantum Mechanics:
# of particle in the ground state

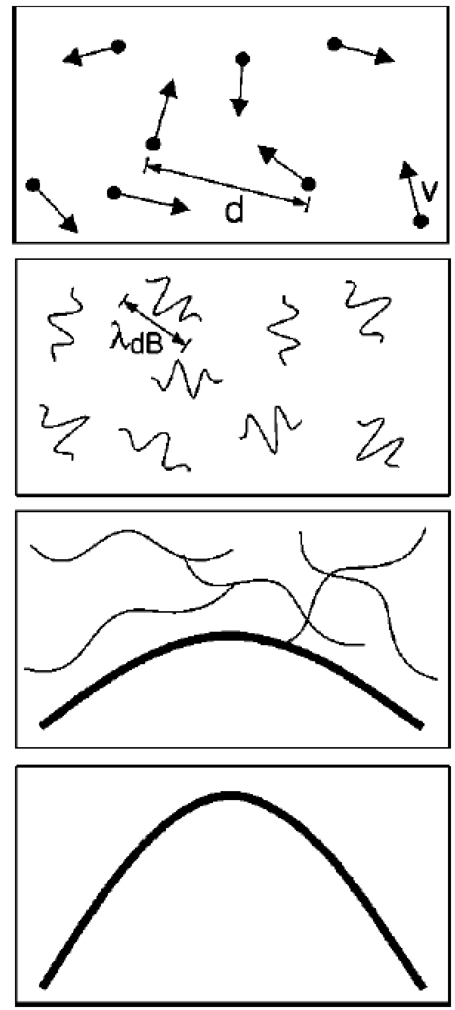
Classical mechanics:
# of particle in a unit phase space

$$\phi = NP(E_0)$$
$$= n\lambda_{dB}^{\ 3}$$
$$\sim Ne^{-H(x,p)/kT}(dxdp/\hbar)^3$$

Magneto-optical trap
Molasses

100 μK ⟷ 10 μK

$\phi = n\lambda^3 = 10^{-7}$



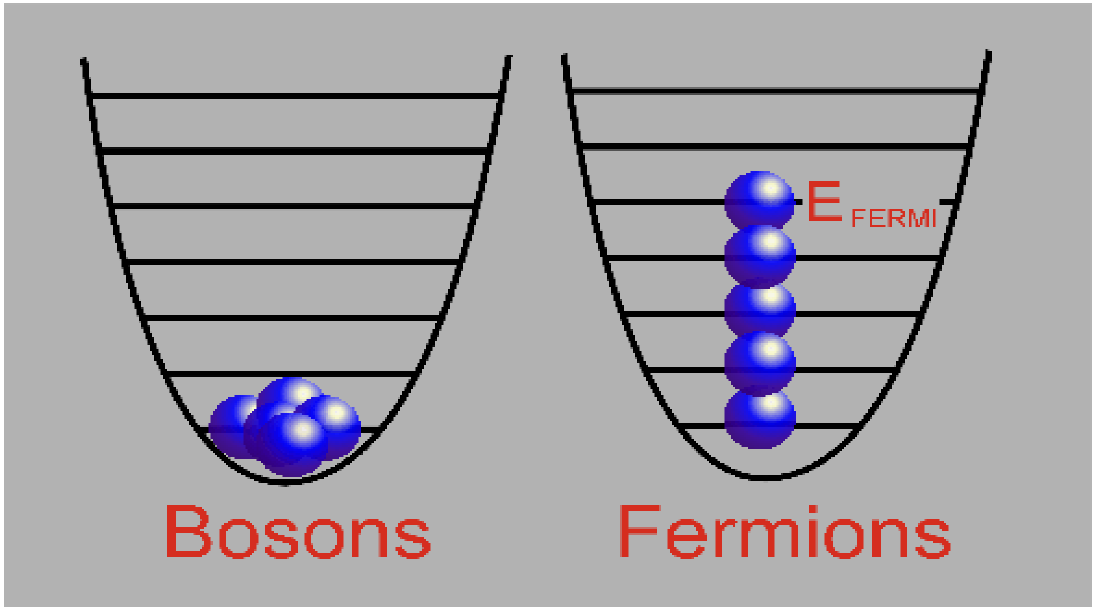
High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"

Low Temperature T:
De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$
"Wave packets"

$T=T_C$:
BEC
$\lambda_{dB} \approx d$
"Matter wave overlap"

T=0:
Pure Bose condensate
"Giant matter wave"

Non-interacting BEC



Atomic wavefunction

$$\Psi(x_1, x_2 \ldots) = \Pi_i \psi_0(x_i)$$

Product state: all atoms occupy ground state.

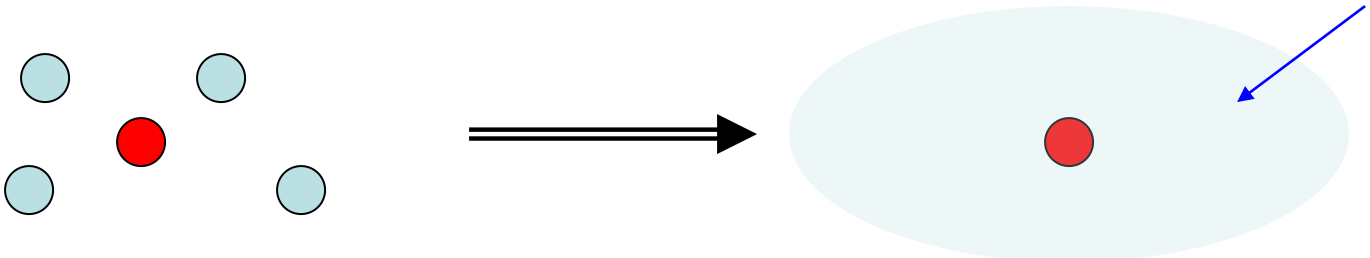
$$E_0\psi_0(r) = -\frac{\hbar^2}{2m}\nabla^2\psi_0(r) + V(r)\psi_0(r)$$

Weakly-interacting BEC: Mean-field approximation

$$V = g\sum_{i<j} v(|r_i - r_j|)$$

$$g\,|\psi_0(x)|^2$$

mean-field



Assume all atoms have the same wave function

$$\mu\psi_1(r) = (-\frac{\hbar^2}{2m}\nabla^2 + V(r) + g\,|\psi_0(r)|^2)\psi_1(r)$$

$$\mu\psi_2(r) = (-\frac{\hbar^2}{2m}\nabla^2 + V(r) + g\,|\psi_1(r)|^2)\psi_2(r)$$

…

A "self-consistent" Gross-Pitaevskii equation

$$\mu\psi(r) = (-\frac{\hbar^2}{2m}\nabla^2 + V(r) + g\,|\psi(r)|^2)\psi(r)$$

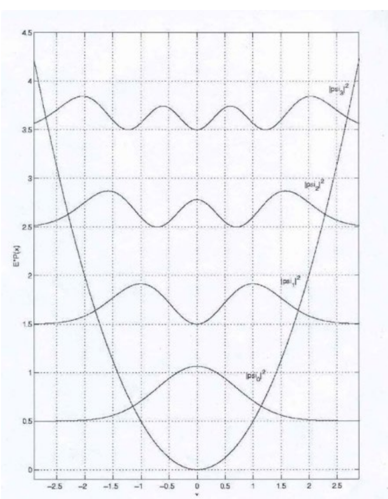
Gross-Pitaevskii equation

$$\mu\Psi(\mathbf{r}) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r}) + V(\mathbf{r})\Psi(\mathbf{r}) + g|\Psi(\mathbf{r})|^2\Psi(\mathbf{r})$$

BEC in free space $\psi(x) = const.$

Chemical potential = zero-point energy + gn $g = \frac{4\pi\hbar^2 a_s}{m}$

BEC in a harmonic trap



Repulsive atomic interaction g>0

- ⇒ increasing mean-field energy
- ⇒ increasing wavefunction spread
- ⇒ increasing potential energy
- ⇒ decreasing kinetic energy

Thomas-Fermi approximation (E_k << V ~ gn)

Thomas-Fermi approximation: KE << int.

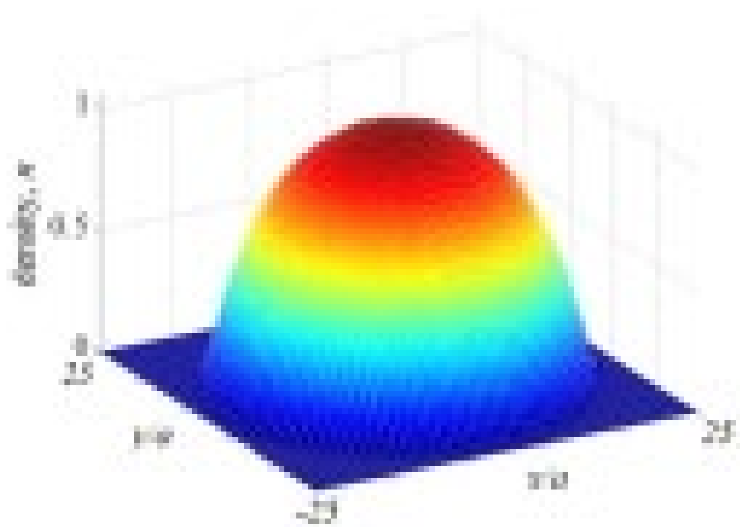
$$\mu\psi(r) = (-\frac{\hbar^2}{2m}\nabla^2 + V(r) + g\,|\psi(r)|^2)\psi(r)$$

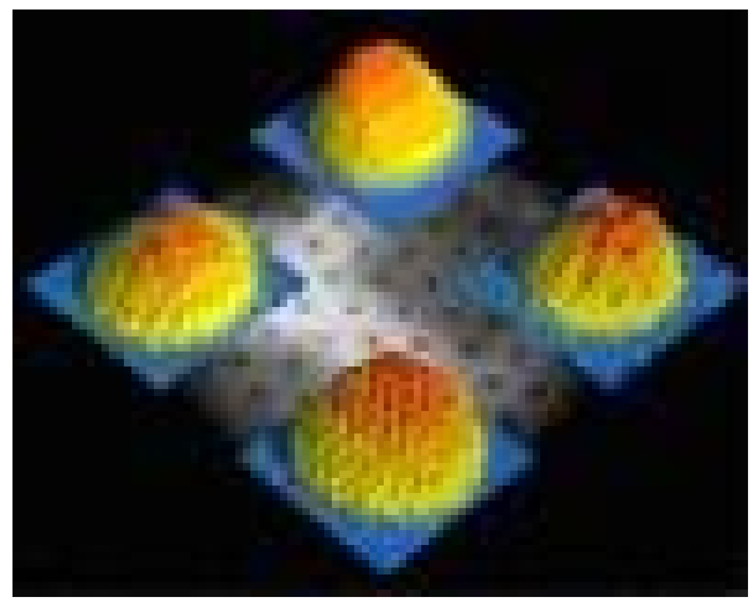
$$\mu\psi(r) = (V(r) + gn)\psi(r)$$

$$n(r) = \frac{\mu - V(r)}{g}$$

For μ >V(r)

$$N = \int n(r)dv$$





Stability of BEC of size L

in a box

Kinetic energy = L^{-2}

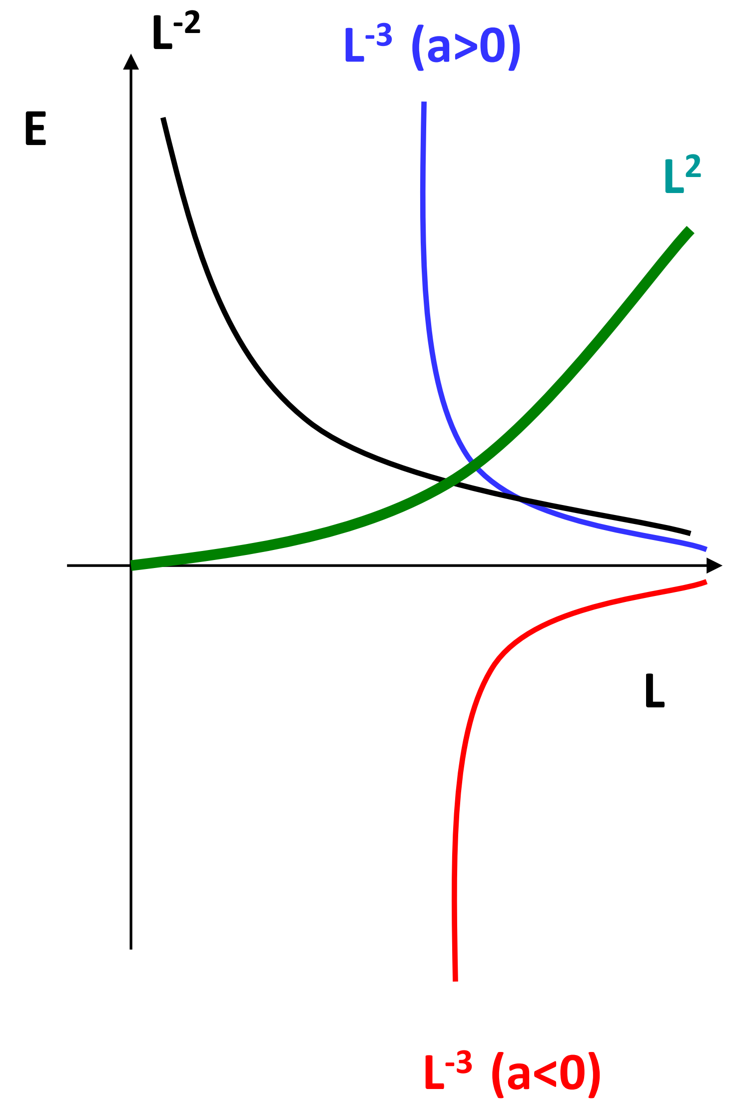
Interaction energy = a N L^{-3}

in a harmonic trap

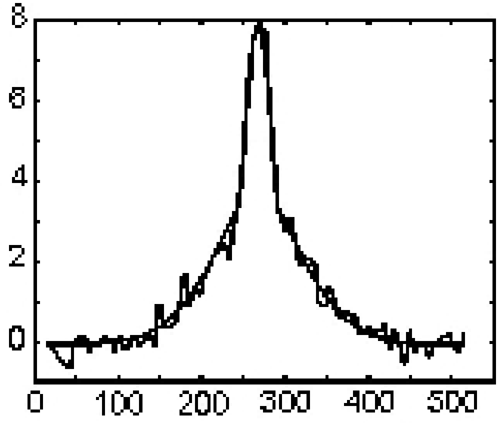
Kinetic energy = L^{-2}

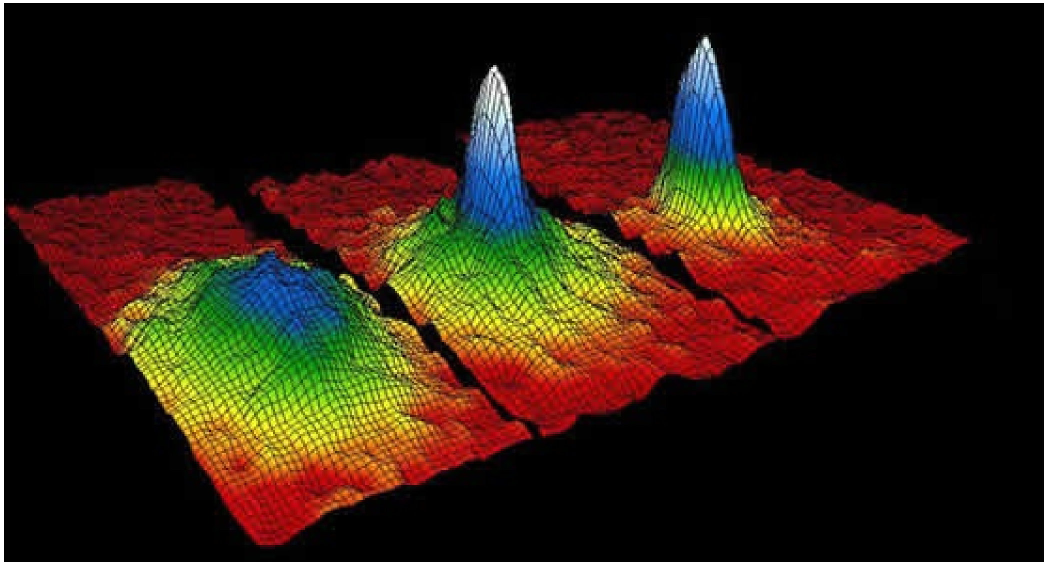
Interaction energy = a N L^{-3}

Potential energy = L^{2}



Finite temperature





Total particle number $N=N_0$ (condensed) $+N'$ (uncondensed)

$$N' = \sum_{i=\mu+1}^{\infty} n_i$$

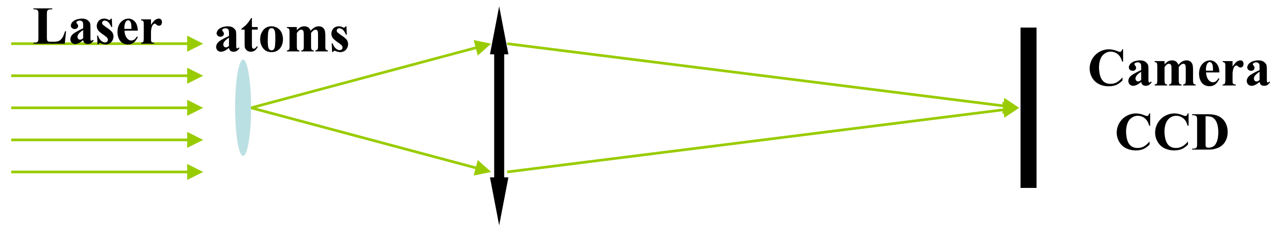
$$= \int d\varepsilon\ \rho(\varepsilon)\frac{1}{e^{\varepsilon/kT}-1}$$

$$= N(T/T_c)^{x+1}$$

$$n_i = \frac{g_i}{e^{(\epsilon_i-\mu)/kT}-1}$$

Density of state: $\rho(\varepsilon) \sim \varepsilon^x$

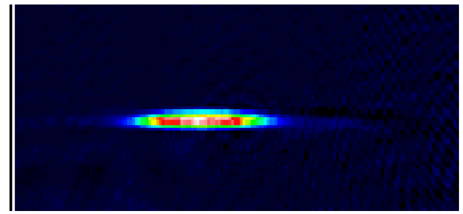
Experiment: Expanding thermal gas and expanding condensate



100 µm * 5 µm

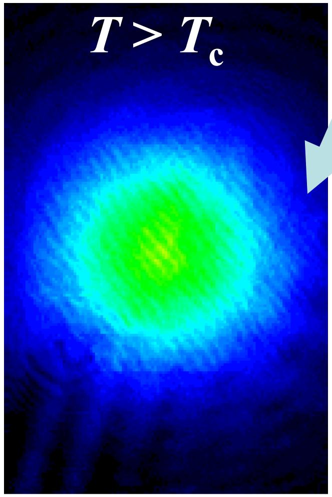
0,5 to 1 µK

3 10⁶ atoms in an anisotropic magnetic trap



Time of flight

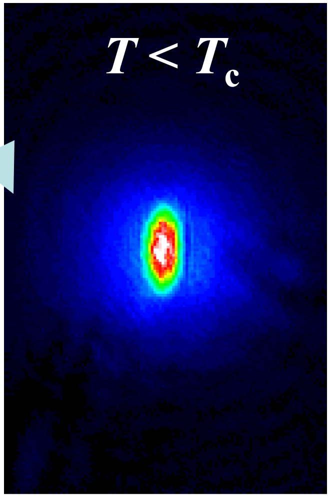
Boltzmann gas



$$\frac{1}{2} m v_i^2 = \frac{1}{2} kT$$

isotropic expansion

condensate

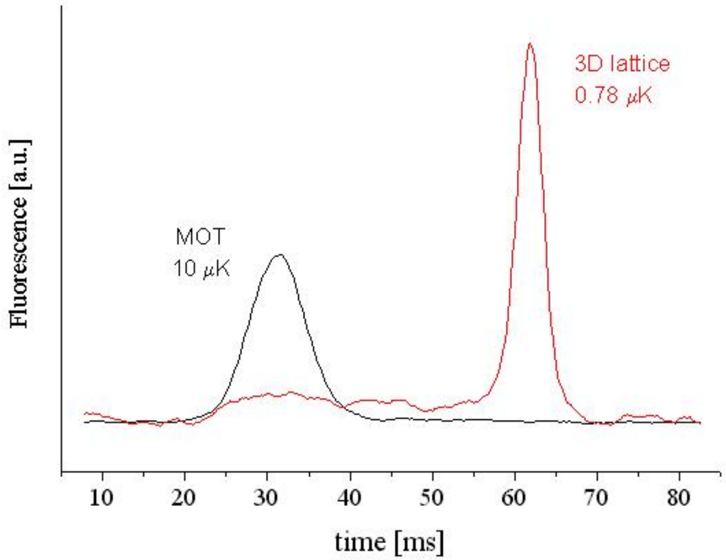


$$\frac{1}{2} m v_i^2 = \frac{1}{4} \hbar \omega_i$$

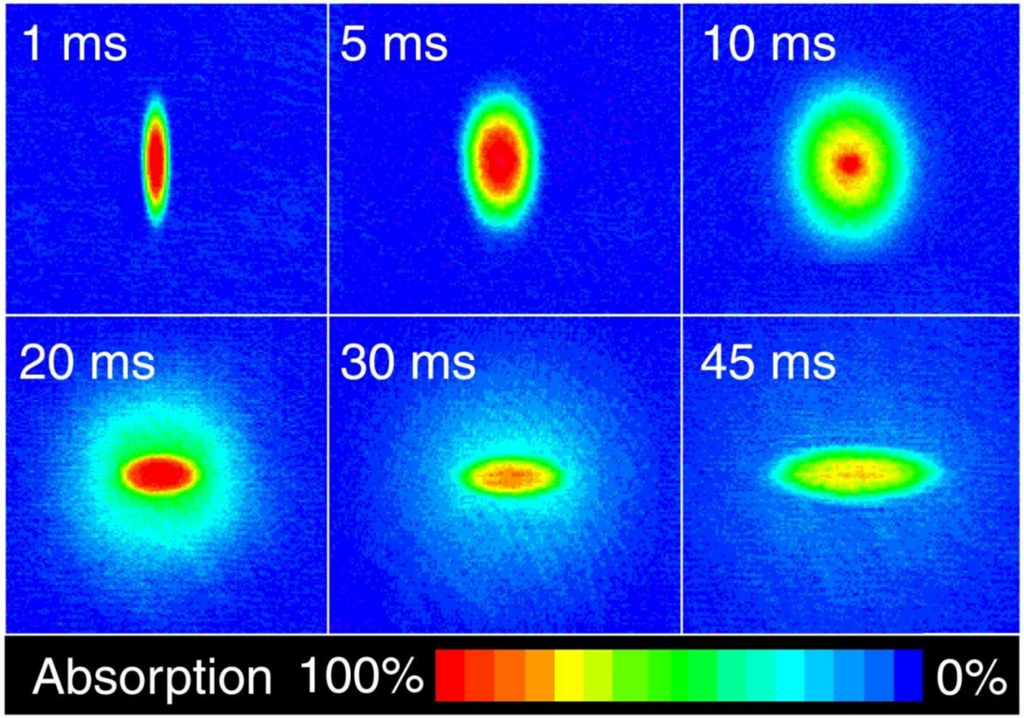
anisotropic expansion

TOF: free expansion of cold gas in vacuum

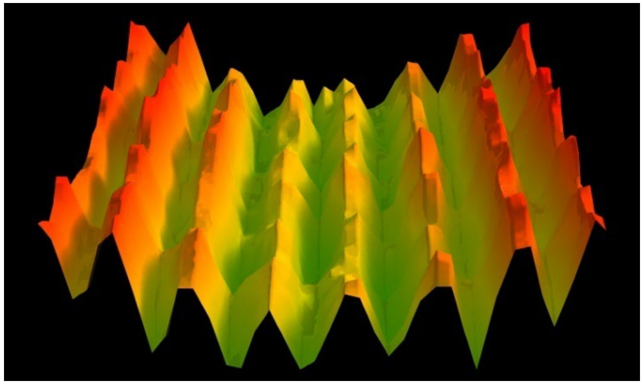
Light sheet



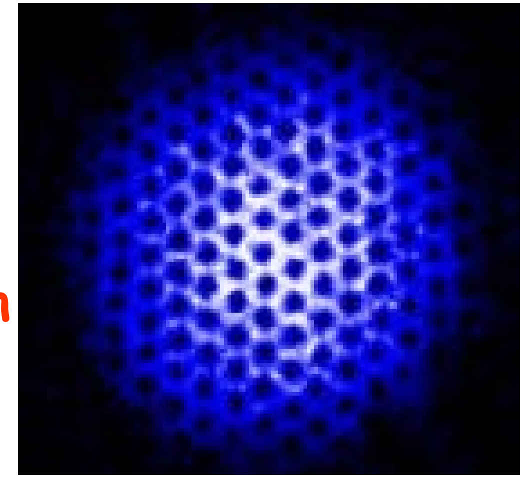
Absorption imaging



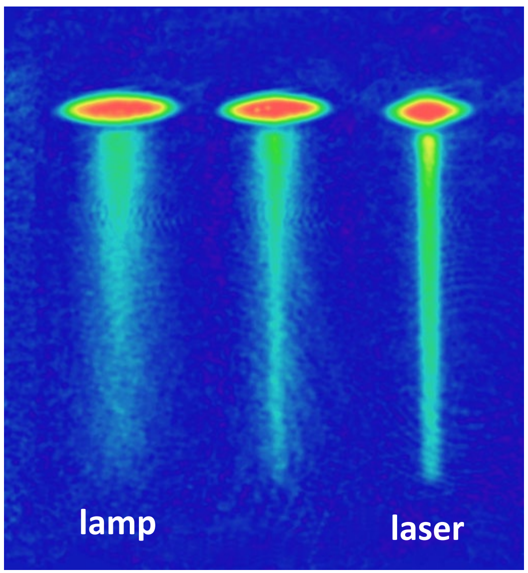
BEC as a coherent source of matterwave (1995~2001)



Matter waves Interference
(MIT group, 1997)

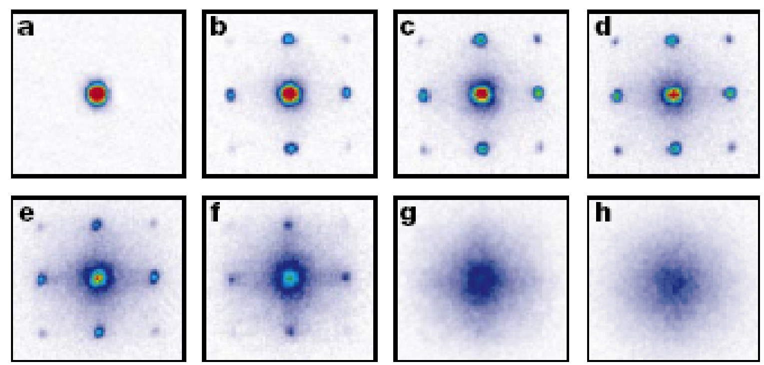


Vortices in Bose-Einstein Condensation
(JILA group, 2000)



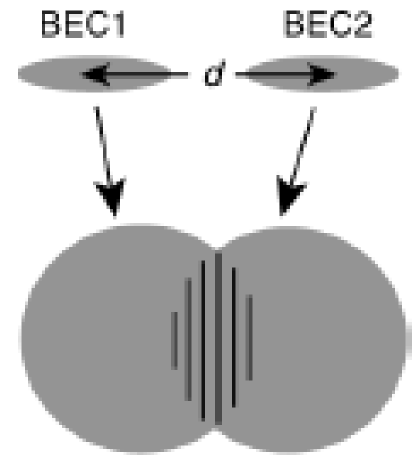
Matter wave laser
(MPQ group, 2000)

Quantum phase transition in optical lattice
(Mainz/MPQ group, 2002)



Trapped BEC's

BEC's after an expansion time t



$$\lambda = \frac{h}{m\Delta v} = \frac{ht}{md}$$

M. R. Andrews *et. al.*
Science 275, ff. 637, 1997