$$H = \int \left(-\varphi^* \frac{\hbar^2}{2m} \nabla^2 \varphi + \frac{\lambda}{2} \varphi^{+2} \varphi \right) dx$$

$$\varphi^+ \varphi (\varphi^+ \varphi - 1)$$

$$\lambda = 4\pi \hbar^2 / a$$

ground state $|g\rangle = |N.0.0.0\dots\rangle$

$$a_0 |g\rangle = \sqrt{N-1}\, |N-1.000\rangle$$

$$a_k |g\rangle = 0$$

$N \to \infty$ $\hat{a}_0 |g\rangle \approx \sqrt{N}\, |N.000\dots\rangle$

$\hat{a}_0$ can be replaced by a #

$\hat{a} |c\rangle = c |c\rangle$ coherent state

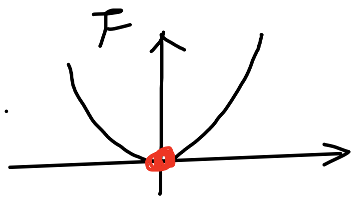
$\Rightarrow$ $\hat{\varphi} = V^{-1/2} \sum \hat{a}_k e^{ikr}$

$\to V^{-1/2} \sum c_k e^{ikr} = \varphi$ classical field.

$$\hat{\varphi} |\varphi_1\rangle = \varphi_1 |\varphi_1\rangle$$

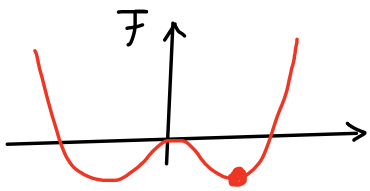
Expectation of free energy $F = \langle \varphi | H - \mu N | \varphi \rangle = \frac{\lambda}{2} |\varphi|^4 - \mu |\varphi|^2$

$\mu < 0$.



ground state $\varphi = 0$

$\mu > 0$



ground state $\varphi = \sqrt{N} e^{i\phi}$

U(1) symmetry breaking