Topics

1. Thermal and classical sample

Collisionless and hydrodynamic regime
Thermal equilibrium

2. Quantum scattering

Elastic and inelastic collisions
Scattering matrix
Scattering cross section
Scattering wave function
Scattering phase shifts and scattering length

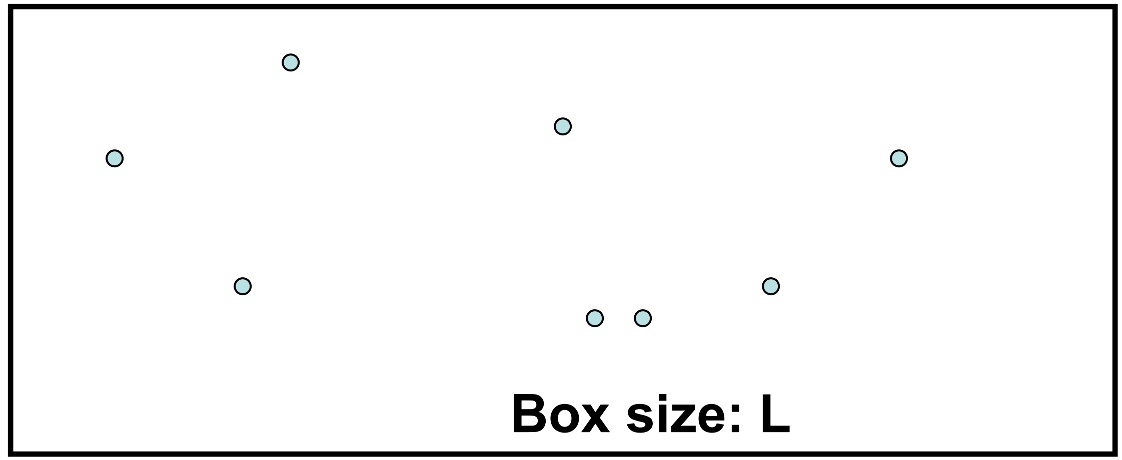
3. Example:

Hard-core potential
Box potential
Resonant scattering

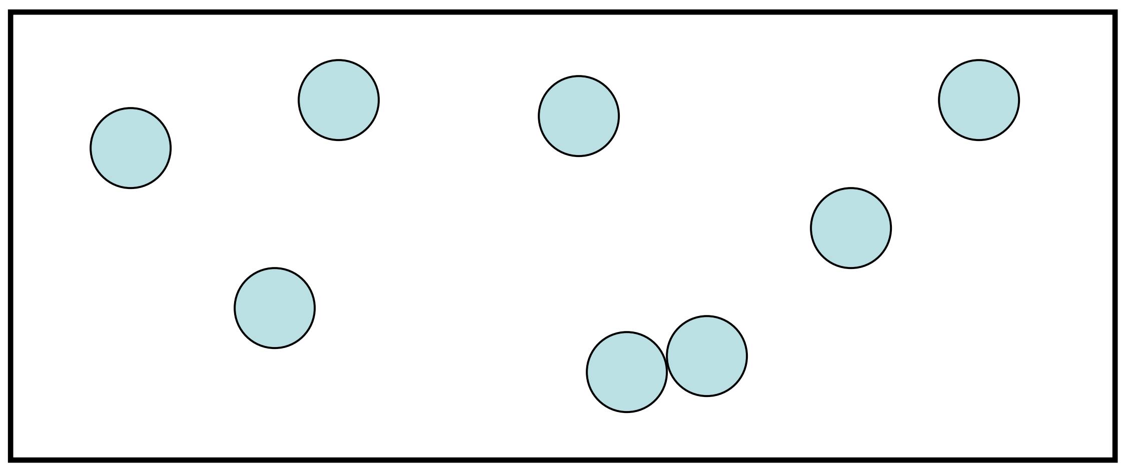
Collisionless and hydrodynamic regime

Mean free path >> L

$1/n\sigma$



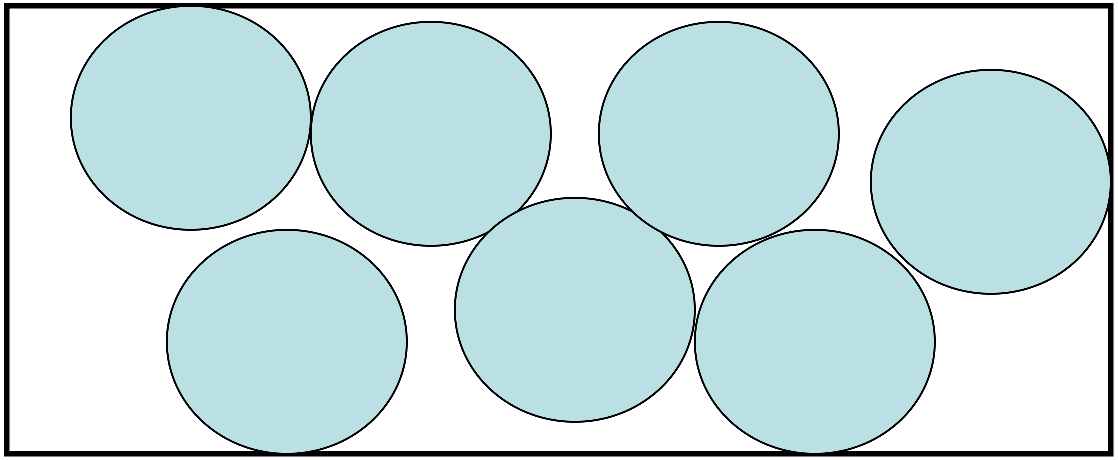
Mean free path << L



Interaction is dominated by two-body collisions

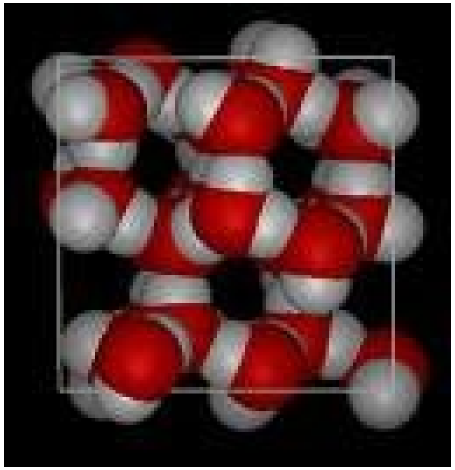
What happen if …

Mean free path << particle size

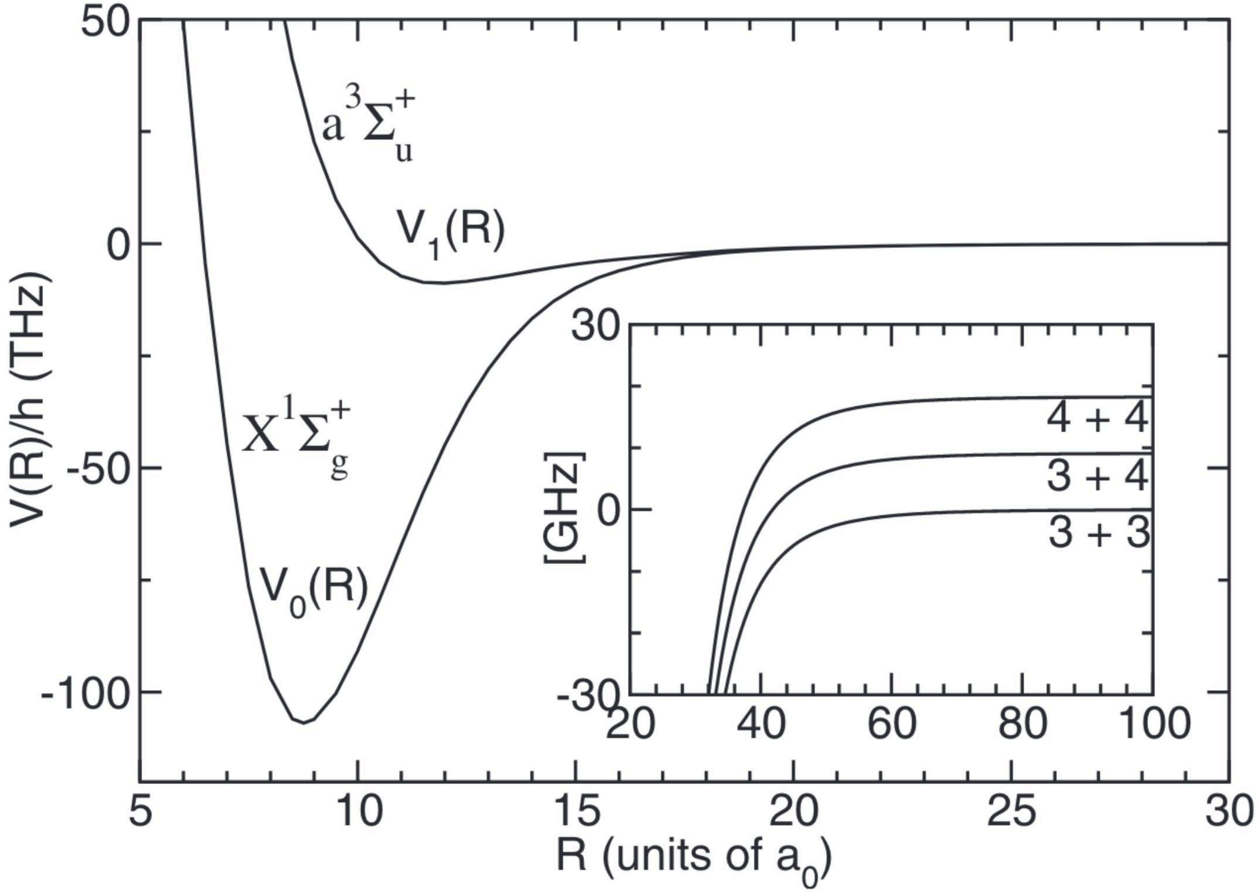


Two-body no longer dominates…

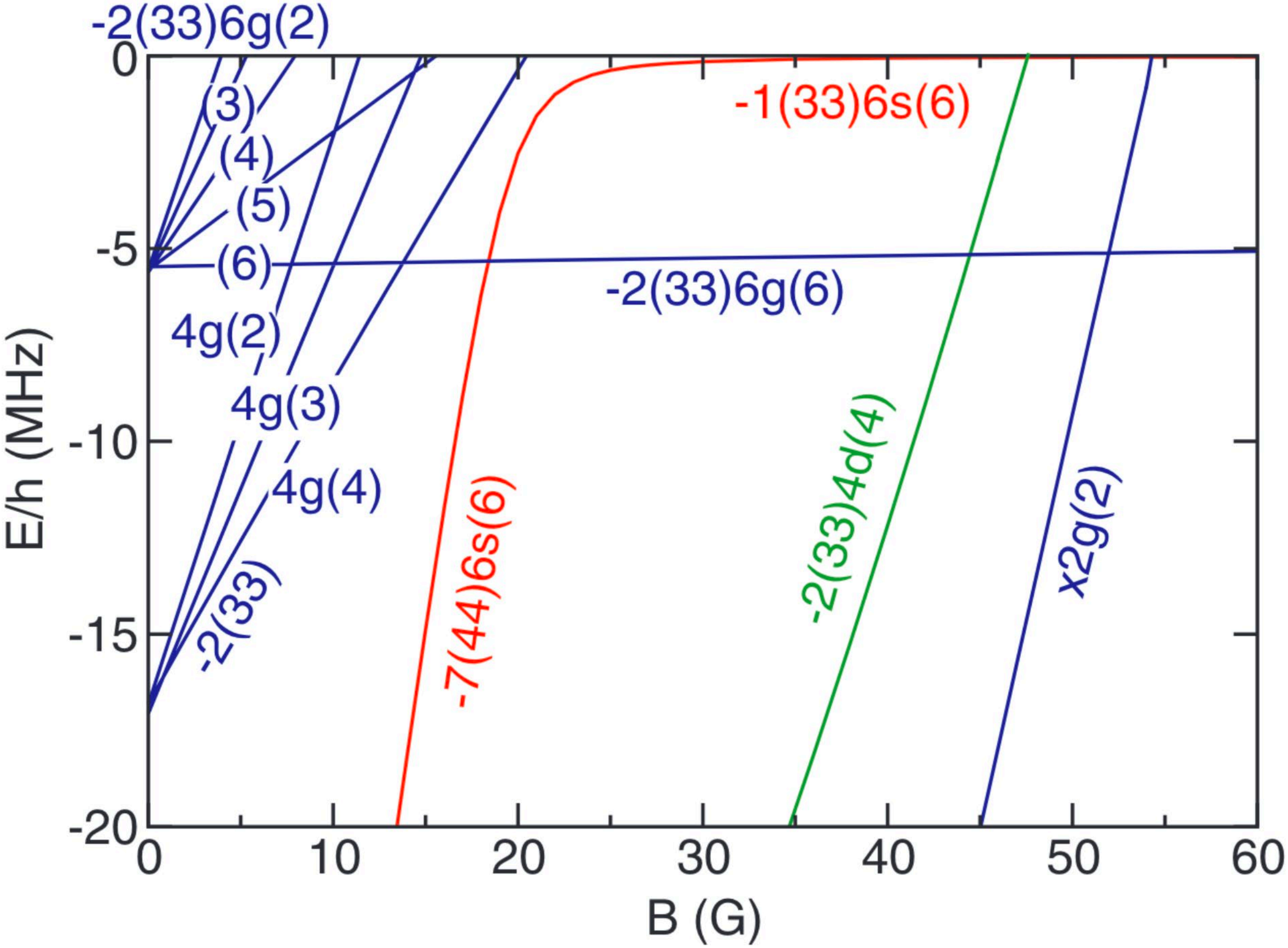
- ***Many-body effects***
- ***Long range order***
- ***Phase transitions***



Molecular potential of Cs2



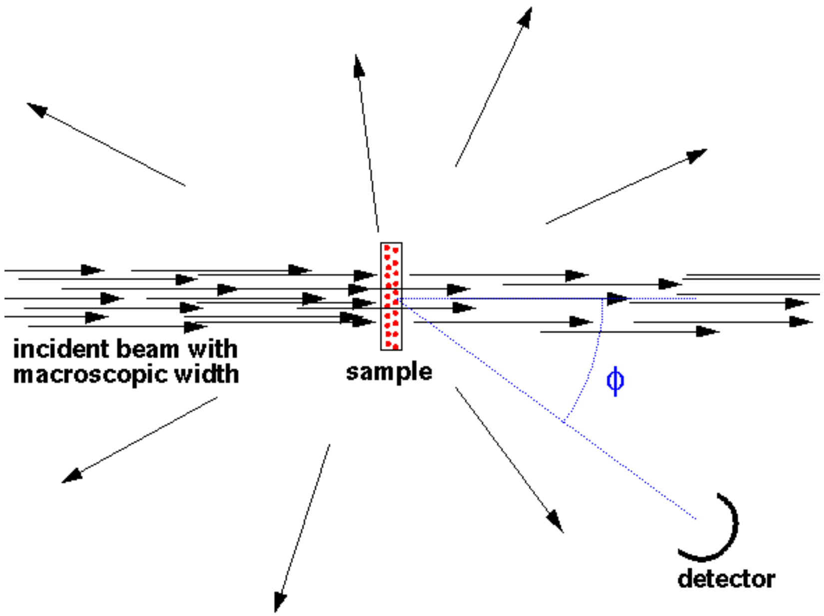
Molecular structure near the continuum



Quantum scattering theory

No scattering

$$|k> = \sum_{\sigma,l} \frac{e^{ikr}}{r} |l> - \frac{e^{-i(kr - l\pi)}}{r} |l>$$



with scattering

$$|k> = \sum_{\sigma,l} \frac{e^{ikr}}{r} S |l> - \frac{e^{-i(kr - l\pi)}}{r} |l>$$

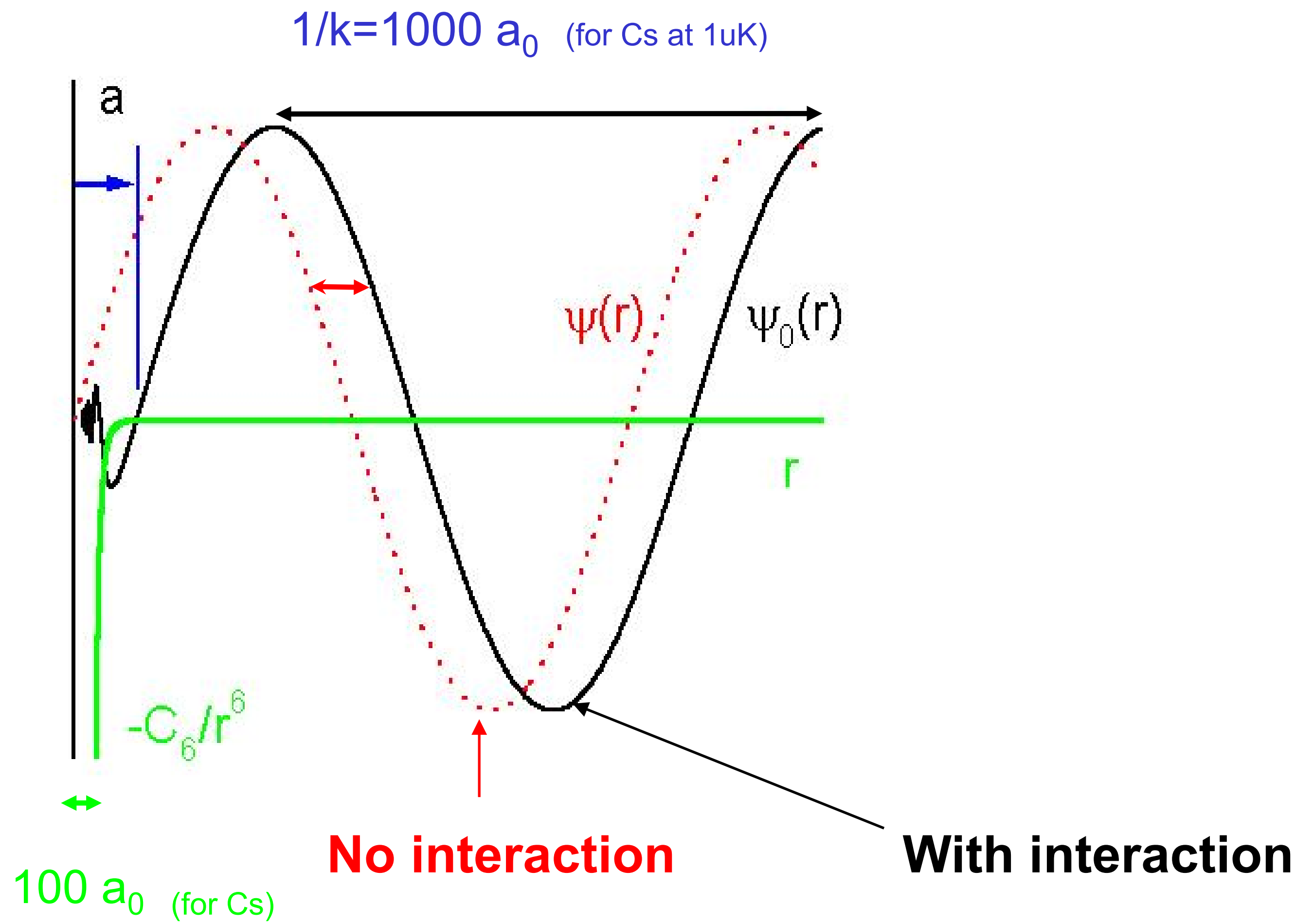
Elastic cross section

$$\sigma_{el} = \frac{\pi}{k^2} \sum_l (2l+1) |1 - S_{ll}|^2$$

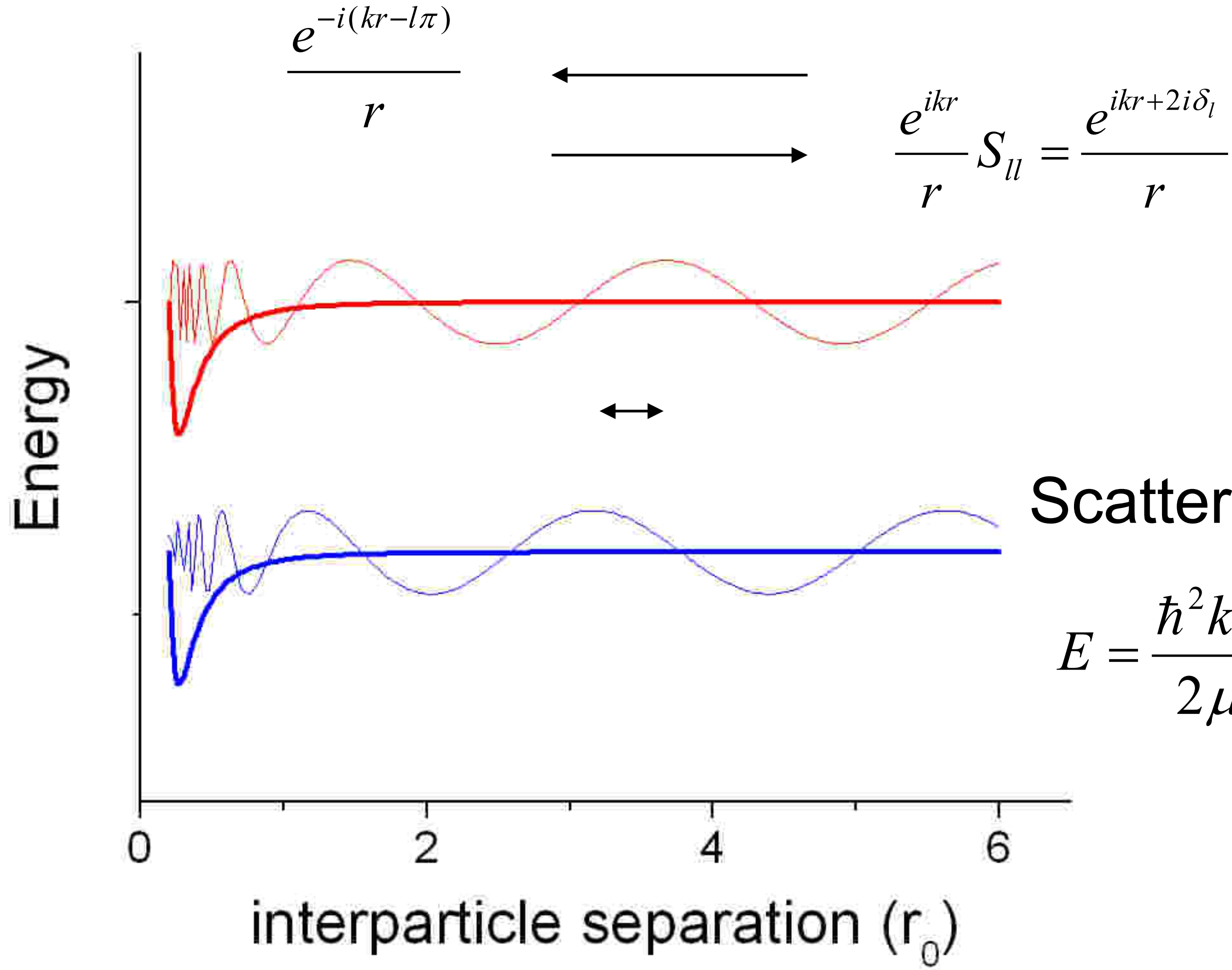
Inelastic cross section

$$\sigma_{in} = \frac{\pi}{k^2} \sum_l (2l+1)(1 - |S_{ll}|^2)$$

Low energy scattering: Only one s-partial wave (spherical wave)



Scattering phase shift



Scattering energy

$$E = \frac{\hbar^2 k^2}{2\mu}$$

Phase shifts δ_l fully determine the scattering

Scattering length

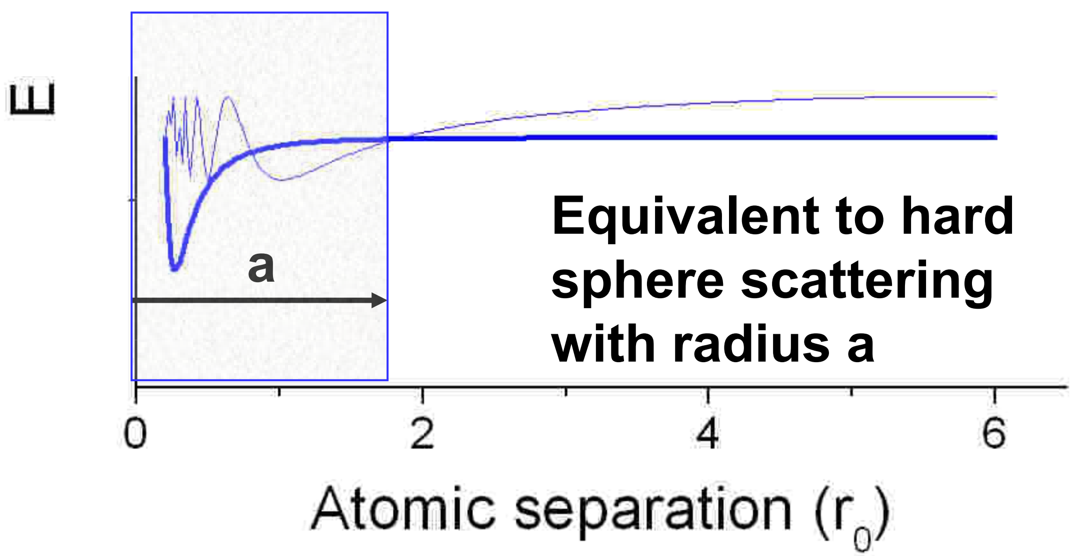
$$\sigma = \frac{4\pi}{k^2}\sum_l (2l+1)\sin^2\delta_l$$

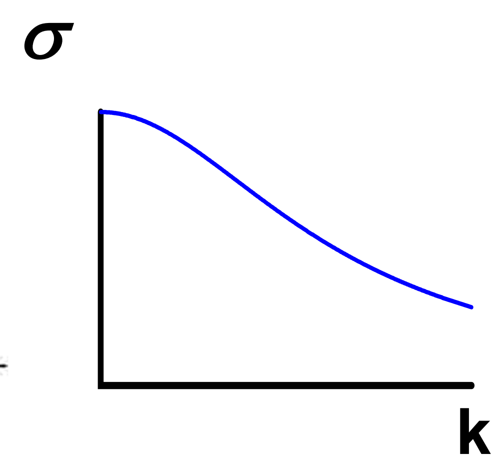
Consider l=0, $$\sigma_0 = \frac{4\pi a^2}{1+k^2a^2} \rightarrow 4\pi a^2$$

Scattering length

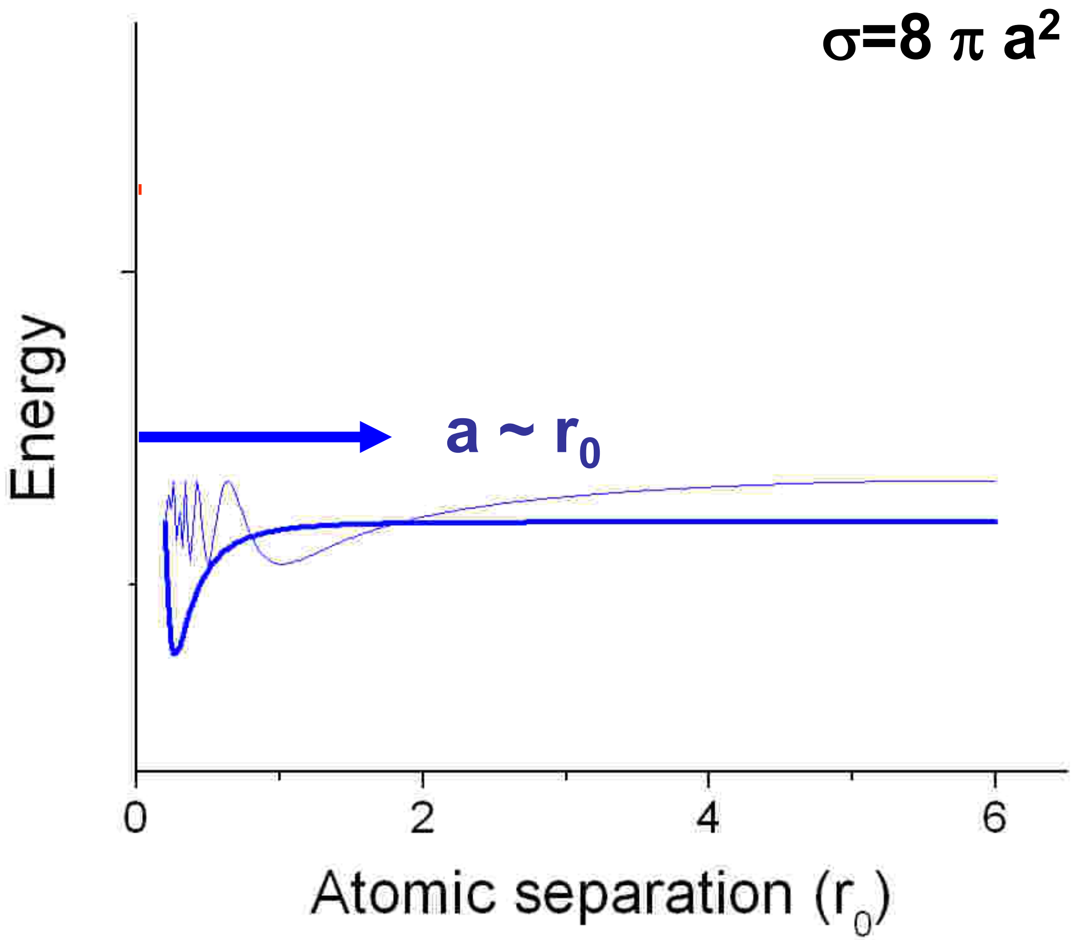
$$k\cot\delta = -\frac{1}{a}$$

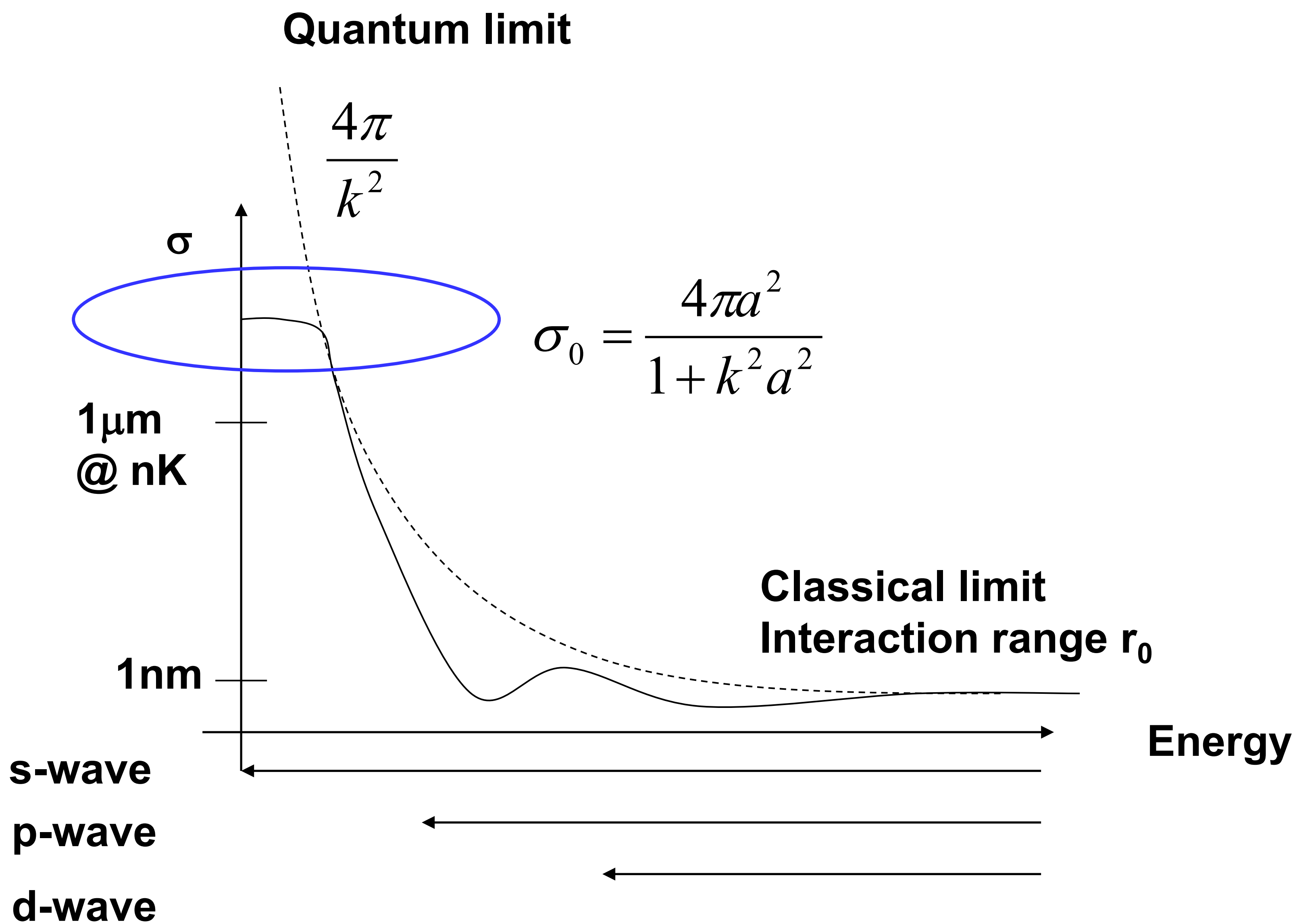
$$a = -\frac{\psi(r>r_0)}{\psi'(r>r_0)}$$



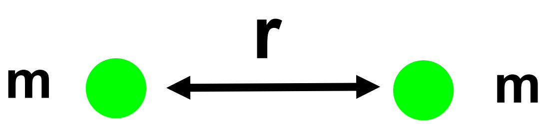


Collisions at low energies

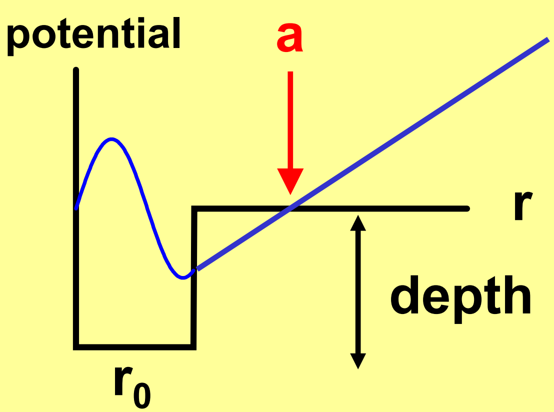



Quantum limit
$\frac{4\pi}{k^2}$
σ
$\sigma_0 = \frac{4\pi a^2}{1+k^2a^2}$
1μm
@ nK
Classical limit
Interaction range r_0
1nm
Energy
s-wave
p-wave
d-wave

Single channel (potential resonance)



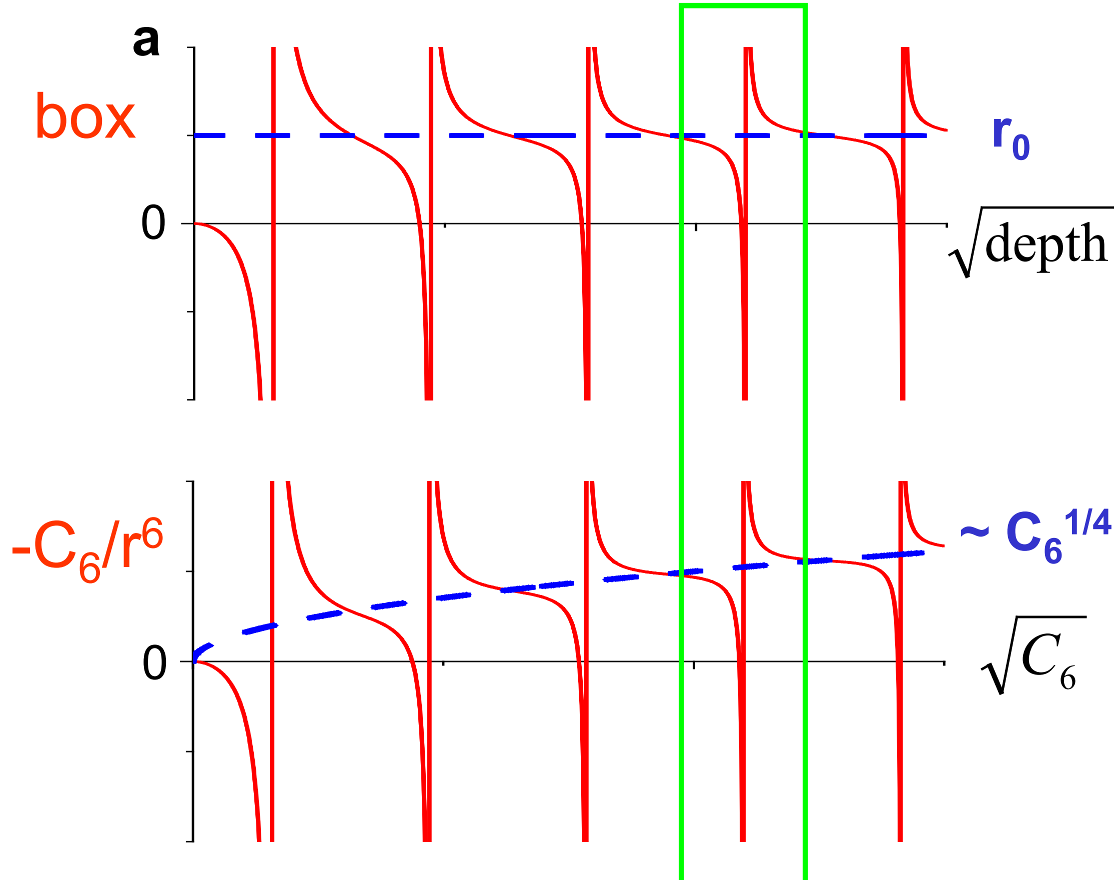
At zero scattering energy



$$\Psi(r<r_0) \sim \frac{\sin q\, r}{r}, \quad \Psi(r>r_0) \sim \frac{r-a}{r}$$

$$\text{B.C.:} \quad \frac{1}{r-a} = \frac{q}{\tan q\, r_0}$$

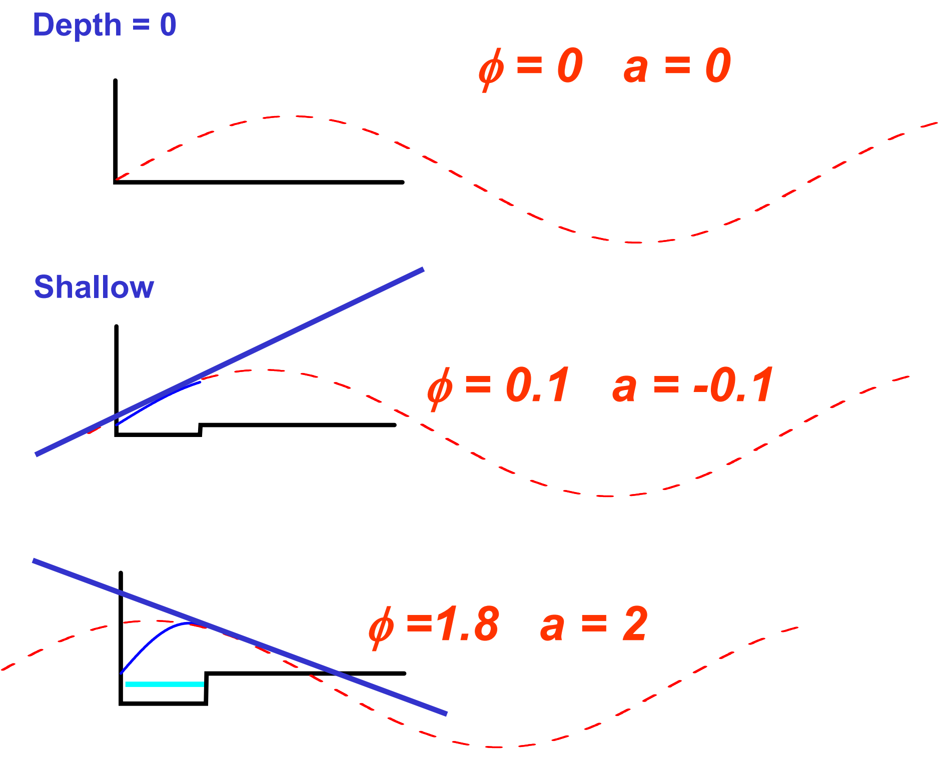
Real atoms: $V(r) = -C_6/r^6$

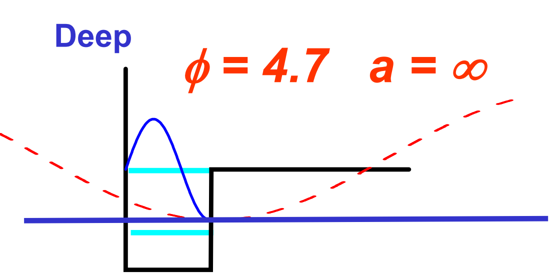


⇒ Effective box size $r_0 = 0.958(mC_6/\hbar^2)^{1/4}$

Identical threshold behavior to real atoms with the effective box potential

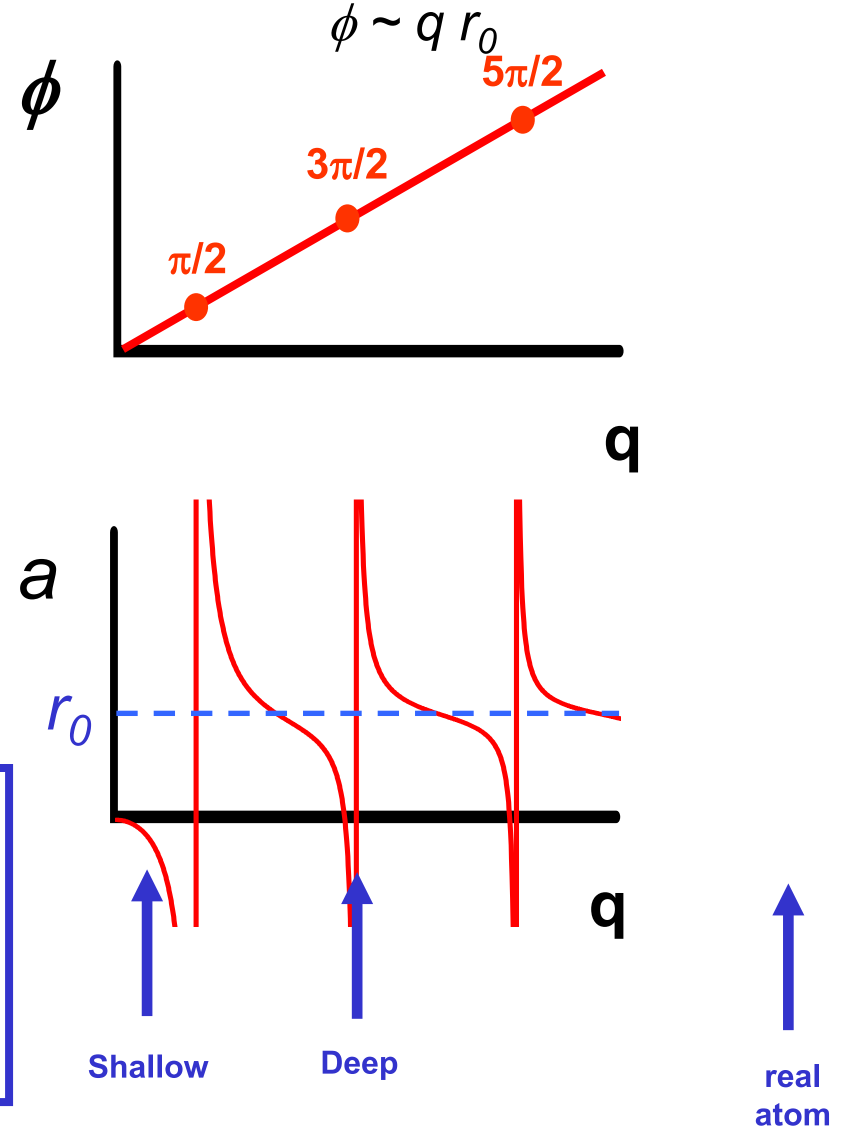
Square well Depth = $\dfrac{\hbar^2 q^2}{m}$ Coll. Energy = $\dfrac{\hbar^2 k^2}{m}$





In general,

$$\frac{1}{r_0 - a} = \frac{k}{\tan q r_0}$$



References

Basic:

Quantum Mechanics, Schiff, pp. 130~133

Modern Quantum Mechanics, Sakurai, pp. 399~418

Bose-Einstein condensation in dilute gases, Pethick, pp. 107~120

Web reference:

http://people.ccmr.cornell.edu/~emueller/scatter/scatter.pdf

Complete

Mott, NF, Massey, HSW (1949). *Theory of Atomic Collisions*. Clarendon Press, Oxford. (1st edition, 1933)