Feshbach resonance P471

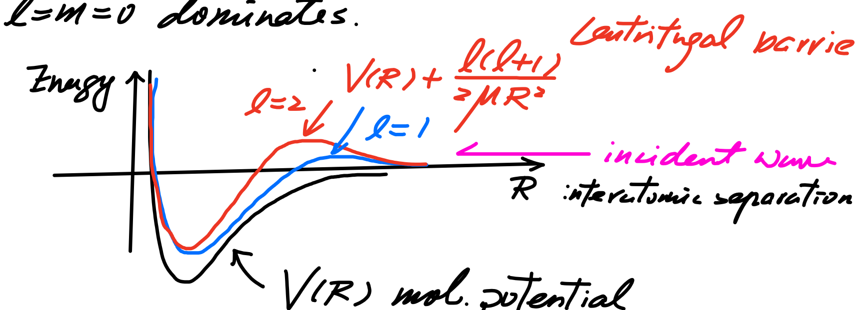
Scattering in QM

Wavefunc $\varphi = \varphi_{inc} + \varphi_s$

$$= e^{ikz} + f(\theta,\phi)\frac{e^{ikr}}{r}$$

$$\equiv e^{ikz} + \sum_{lm} f_{lm} Y_{lm} \frac{e^{ikr}}{r}$$

For low energy scattering, only s-wave scattering with $l=m=0$ dominates.



$(k \to 0)$

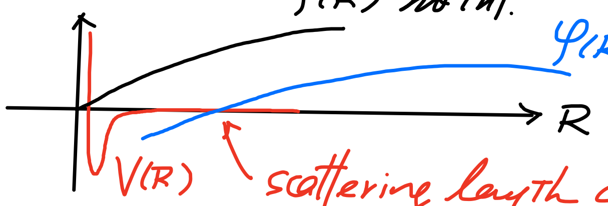
When incident energy $\ll 100\mu K$, only s-wave is left.

Consider only s-wave $e^{ikz} = \frac{e^{-ikr}}{r} - \frac{e^{ikr}}{r} + (l \neq 0 \text{ terms})$

Including scatterer $\varphi = \frac{1}{r}\left[e^{-ikr} - S e^{ikr}\right]$

S is the scattering matrix $S \equiv e^{i2\delta}$

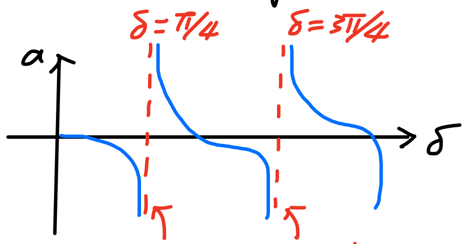
$\Rightarrow \varphi = \frac{1}{r}\left[1 - e^{i2\delta} e^{2ikr}\right] = \frac{-2i}{r} \sin(kr+\delta)$



In the limit of long wavelength $k \to 0$. $\varphi \to \sin k(r-a)$
$\simeq r-a$

$\Rightarrow \lim_{k\to 0} \delta = -ka$

More rigorously $a \equiv \lim_{k\to 0} -\frac{1}{k\cot\delta}$



$\Rightarrow$ Divergence of a is linked to new bound state in the mol. potential.

This is also called potential resonance.