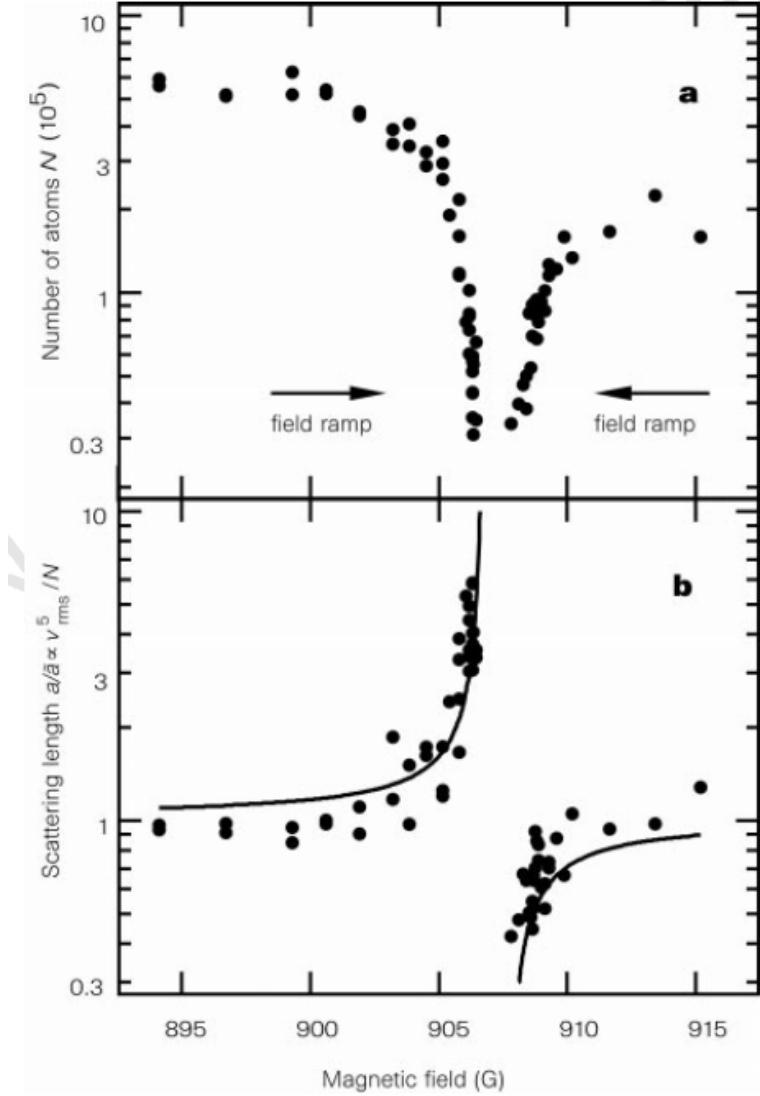


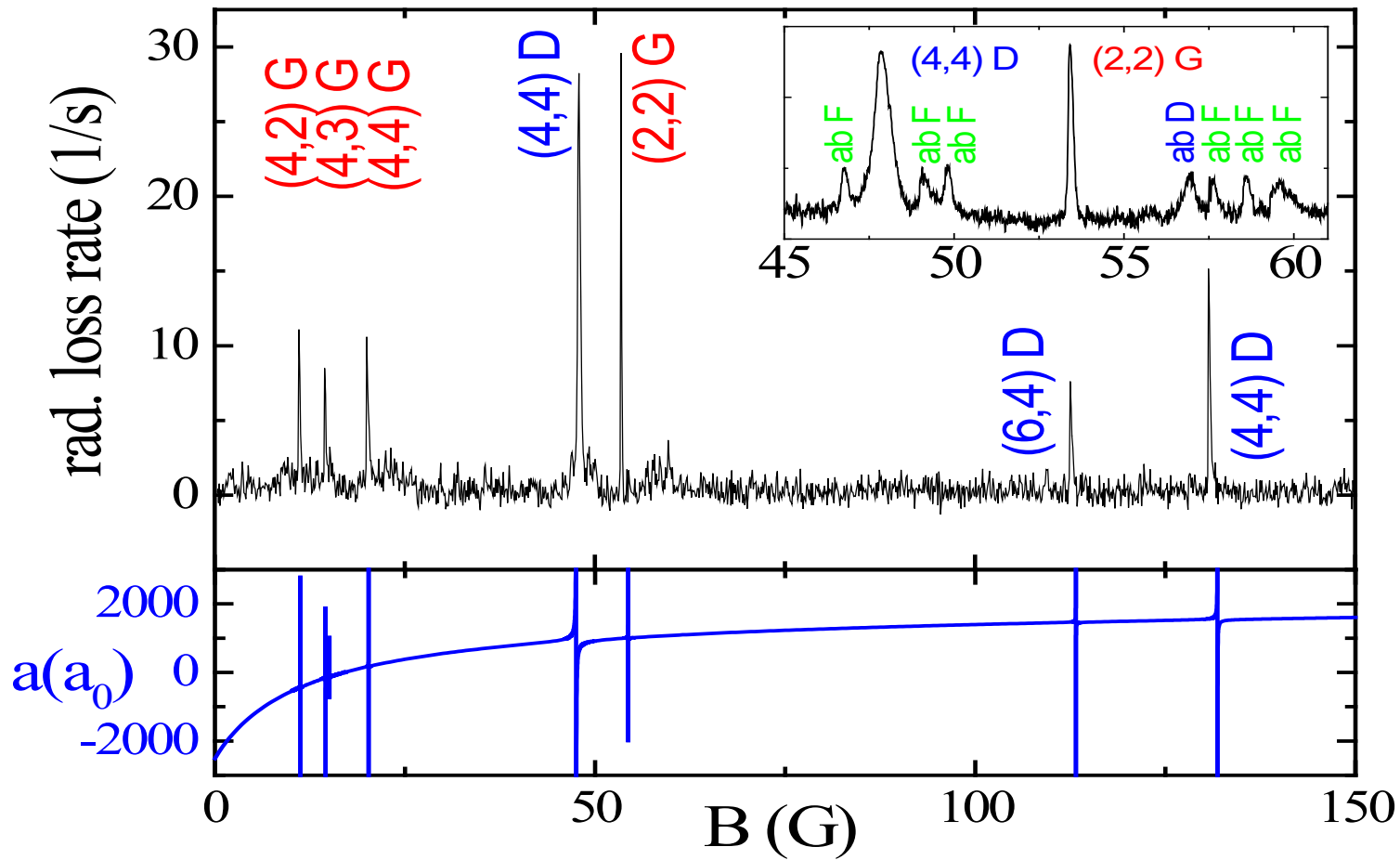
Feshbach resonance in sodium Bose-Einstein condensate (1998 MIT)



Strong inelastic loss

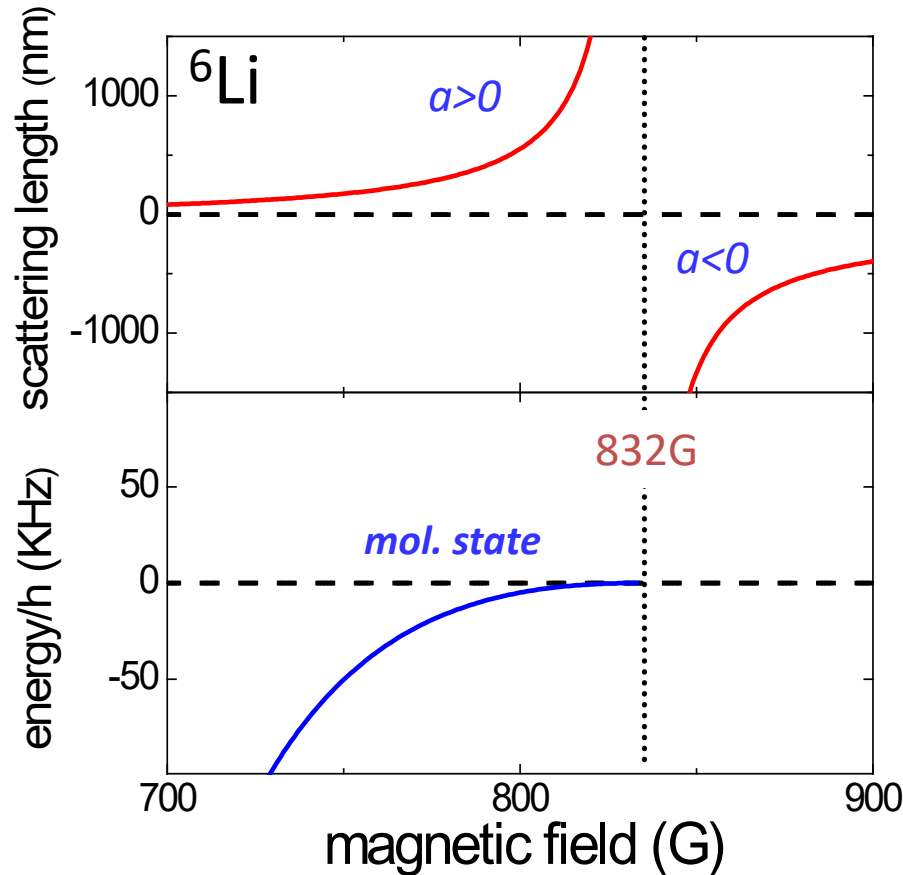
$$a = a_{bg} \left(1 - \frac{\Delta}{B - B_0} \right)$$

Radiative Feshbach resonances in Cesium



Stanford (exp.) and NIST (theo.),
1999-01

Universal behavior in the threshold regime



$$a = a_{bg} \left(1 - \frac{\Delta B}{B - B_0} \right)$$

$$E_b = \frac{\hbar^2}{m(a - r_0)^2}$$

What is scattering length?

Why does molecular state approach continuum non-linearly?

Many more questions...

Feshbach resonances in cold atom collisions

1276

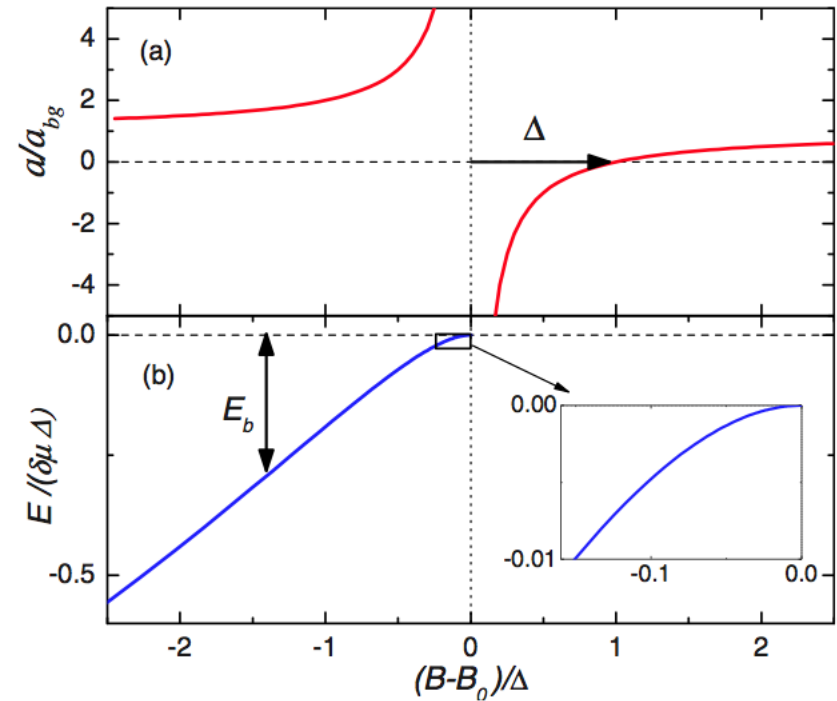
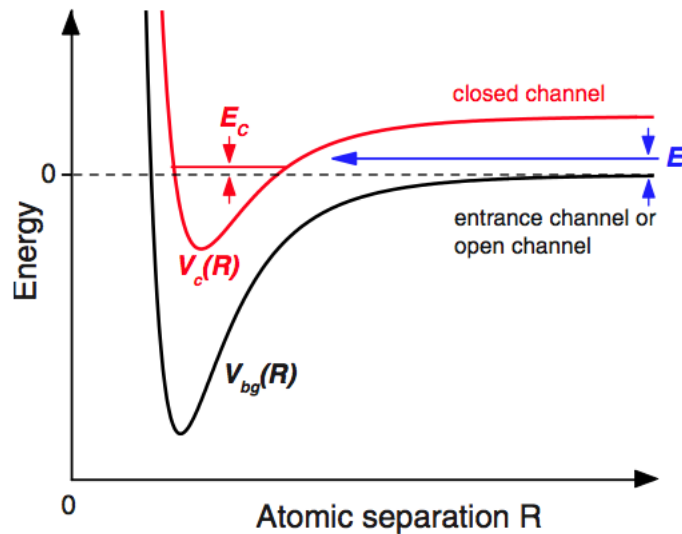
Chin *et al.*: Feshbach resonances in ultracold gases

TABLE IV. Properties of selected Feshbach resonances. The first column describes the atomic species and isotope. The next three columns characterize the scattering and resonance states, which include the incoming scattering channel (ch.), partial wave ℓ , and the angular momentum of the resonance state ℓ_c . This is followed by the resonance location B_0 , the width Δ , the background scattering length a_{bg} , the differential magnetic moment $\delta\mu$, the dimensionless resonance strength s_{res} , the background scattering length in van der Waals units $r_{bg} = a_{bg}/\bar{a}$, and the bound state parameter ζ from Eq. (52). Here a_0 is the Bohr radius and μ_B is the Bohr magneton. Definitions are given in Sec. II. The last column gives the source. A string “na” indicates that the corresponding property is not defined. For example a_{bg} is not defined for p -wave scattering.

Atom	ch.	ℓ	ℓ_c	B_0 (G)	Δ (G)	a_{bg}/a_0	$\delta\mu/\mu_B$	s_{res}	r_{bg}	ζ	Reference
^6Li	<i>ab</i>	<i>s</i>	<i>s</i>	834.1	-300	-1405	2.0	59	-47	1400	Bartenstein <i>et al.</i> , 2005
	<i>ac</i>	<i>s</i>	<i>s</i>	690.4	-122.3	-1727	2.0	29	-58	850	Bartenstein <i>et al.</i> , 2005
	<i>bc</i>	<i>s</i>	<i>s</i>	811.2	-222.3	-1490	2.0	46	-50	1200	Bartenstein <i>et al.</i> , 2005
	<i>ab</i>	<i>s</i>	<i>s</i>	543.25	0.1	60	2.0	0.001	2.0	0.001	Strecker <i>et al.</i> , 2003
	<i>aa</i>	<i>p</i>	<i>p</i>	159.14	na	na	2.0	na	na	na	Zhang <i>et al.</i> , 2004; Schunck <i>et al.</i> , 2005
	<i>ab</i>	<i>p</i>	<i>p</i>	185.09	na	na	2.0	na	na	na	Zhang <i>et al.</i> , 2004; Schunck <i>et al.</i> , 2005
	<i>bb</i>	<i>p</i>	<i>p</i>	214.94	na	na	2.0	na	na	na	Zhang <i>et al.</i> , 2004; Schunck <i>et al.</i> , 2005
^7Li	<i>aa</i>	<i>s</i>	<i>s</i>	736.8	-192.3	-25	1.93	0.80	-0.79	0.31	Strecker <i>et al.</i> , 2002; Pollack <i>et al.</i> , 2009 ^a
^{23}Na	<i>cc</i>	<i>s</i>	<i>s</i>	1195	-1.4	62	-0.15	0.0050	1.4	0.004	Inouye <i>et al.</i> , 1998; Stenger <i>et al.</i> , 1999 ^a
	<i>aa</i>	<i>s</i>	<i>s</i>	907	1	63	3.8	0.09	1.5	0.07	Inouye <i>et al.</i> , 1998; Stenger <i>et al.</i> , 1999 ^a
	<i>aa</i>	<i>s</i>	<i>s</i>	853	0.0025	63	3.8	0.0002	1.5	0.0002	Inouye <i>et al.</i> , 1998; Stenger <i>et al.</i> , 1999 ^a

Reference: Cheng Chin, Rudolf Grimm, Paul Julienne, Eite Tiesinga, RMP (2012)

Physics picture of Feshbach resonance



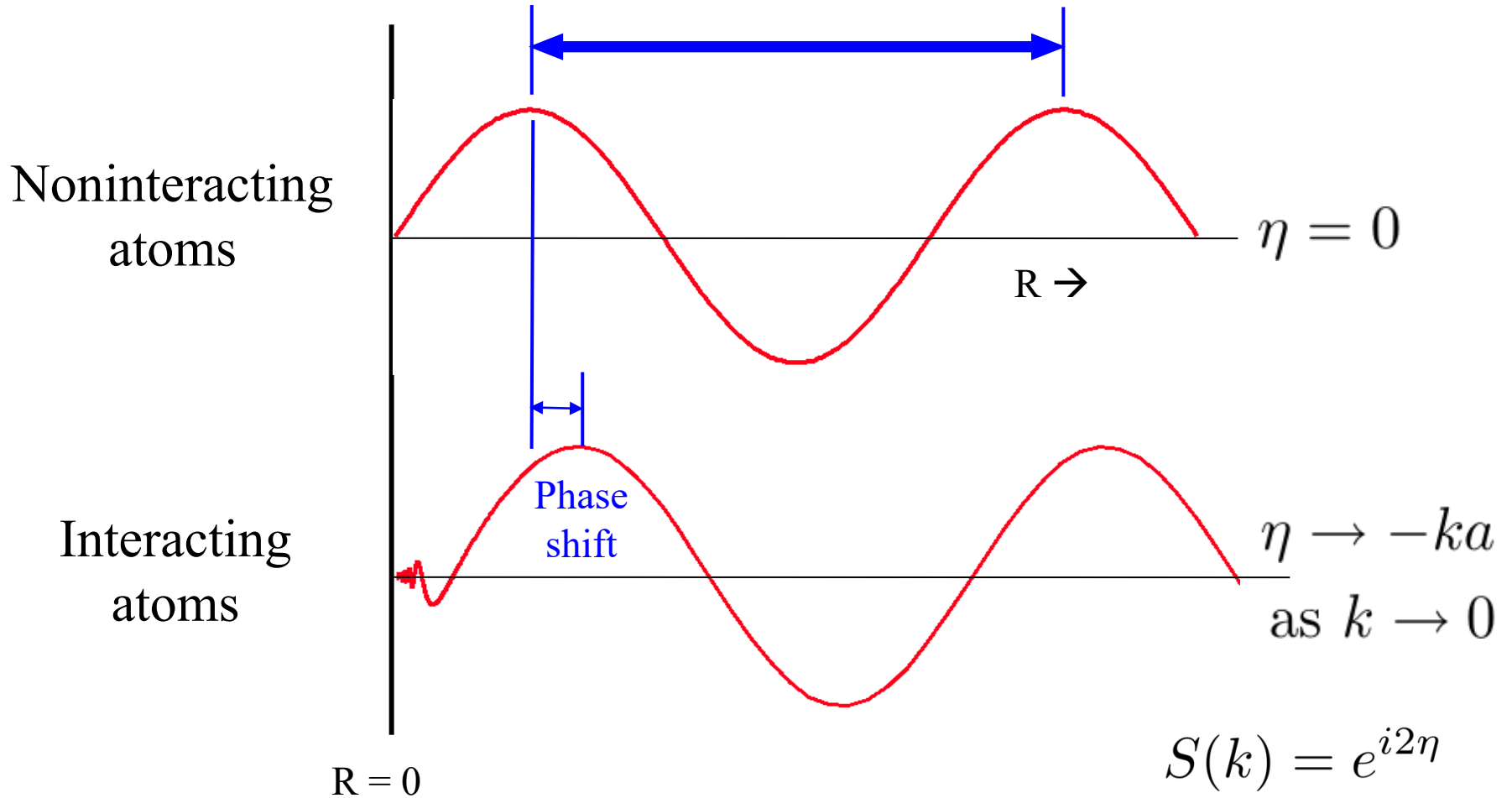
CC, Grimm, Tiesinga, Julienne, RMP (2010)

Simple picture: Feshbach resonance occurs when a bound state in the closed channel matches the scattering state.

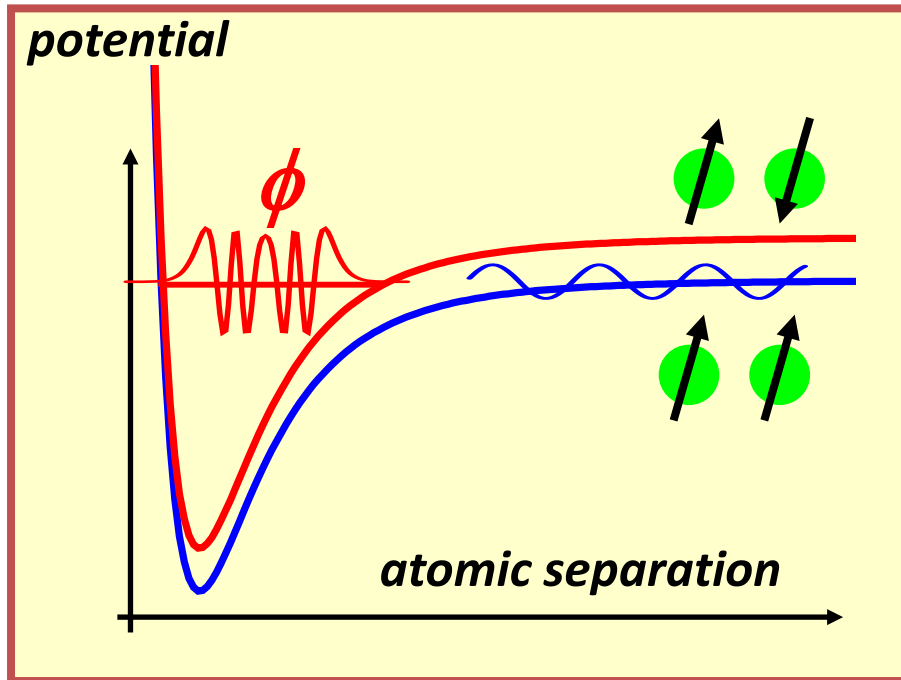
s-wave scattering phase shift

$$\Psi(R) \rightarrow \sin(kR + \eta)$$

Wavelength $\lambda = 2\pi/k$



Scattering channels and Feshbach resonance



Open channel
(typically) Triplet potential

Closed channel
(typically) Singlet potential

Feshbach tuning
External magnetic field

Transition matrix

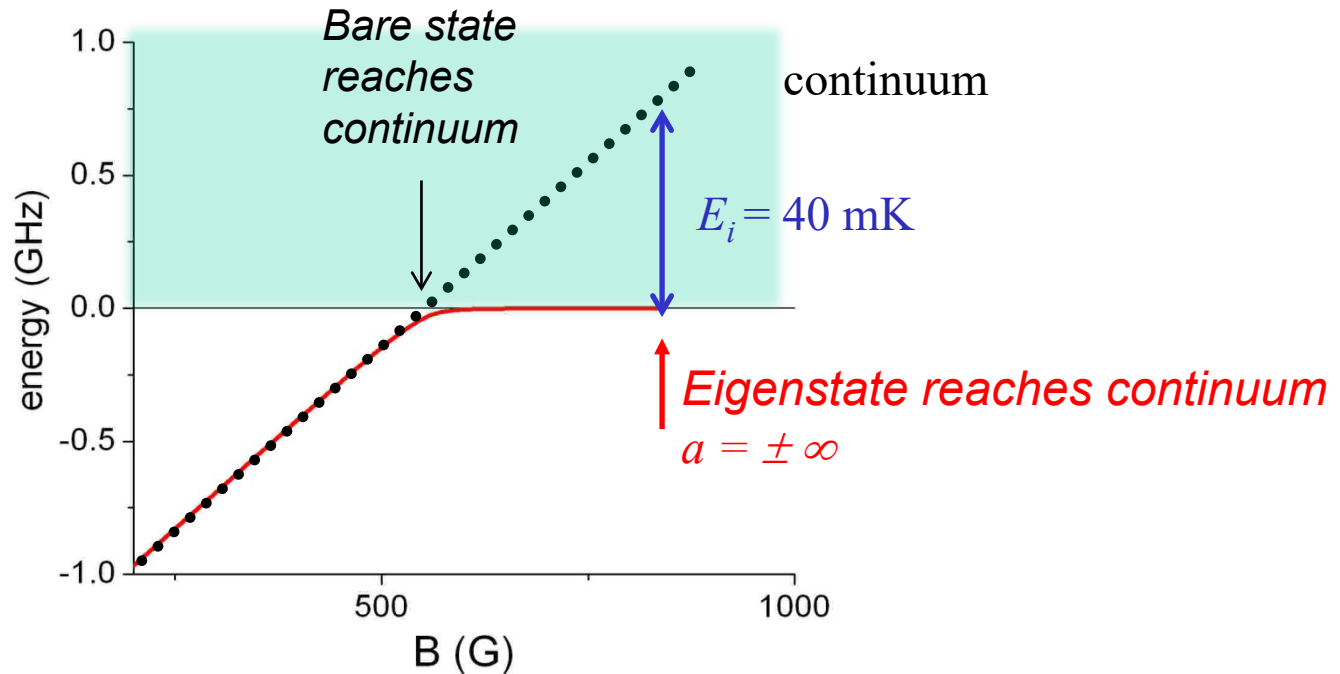
$$T_{fi} = T_{fi}^0 + \frac{\langle \chi_f^- | V | \phi \rangle \langle \phi | V | \chi_i^+ \rangle}{E - E_\phi + i\Gamma/2}$$



Scattering length:

$$a = a_{bg} \left(1 - \frac{\Delta B}{B - B_0} \right)$$

An extreme case: ${}^6\text{Li}$ with $a_{bg} = -1700$ Bohr



$$\frac{1}{a - r_0} = \frac{1}{a_{bg} - r_0} + \frac{\Gamma / 2r_0}{E_i}$$

$$r_0 = 30 a_0$$

E_i = bare state energy

Γ = Feshbach coupling

Simple two-channel model for Feshbach resonance

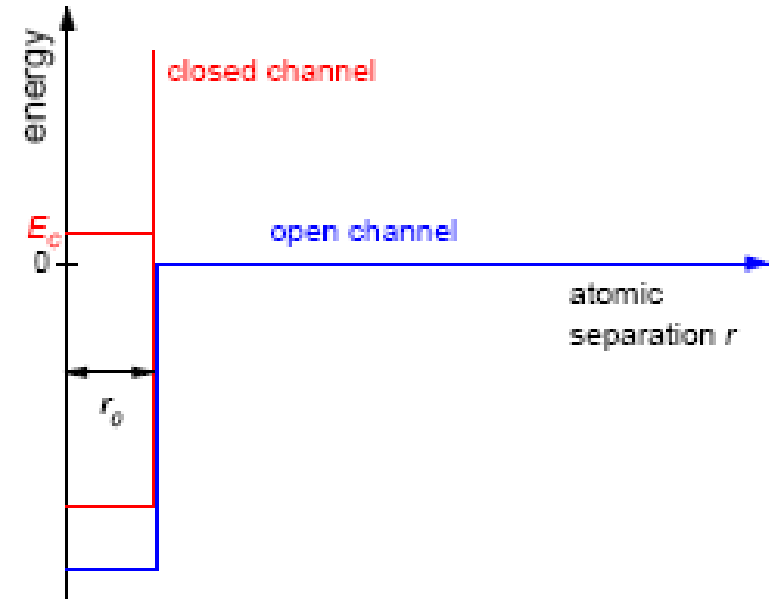
1. Box model with $r_0 \sim r_{\text{vdw}}$ simulates molecular potential
2. Closed channel supports a bound state at E_c near the continuum.

Parameterization

Open channel depth V_o determines a_{bg}

Closed channel depth V_c determines E_c

Feshbach Coupling is Γ .



$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \hat{V}\right)|\psi\rangle = E|\psi\rangle$$

$$\hat{V} = \begin{pmatrix} -V_c & \hbar\Omega \\ \hbar\Omega & -V_o \end{pmatrix} \text{ for } R < a$$

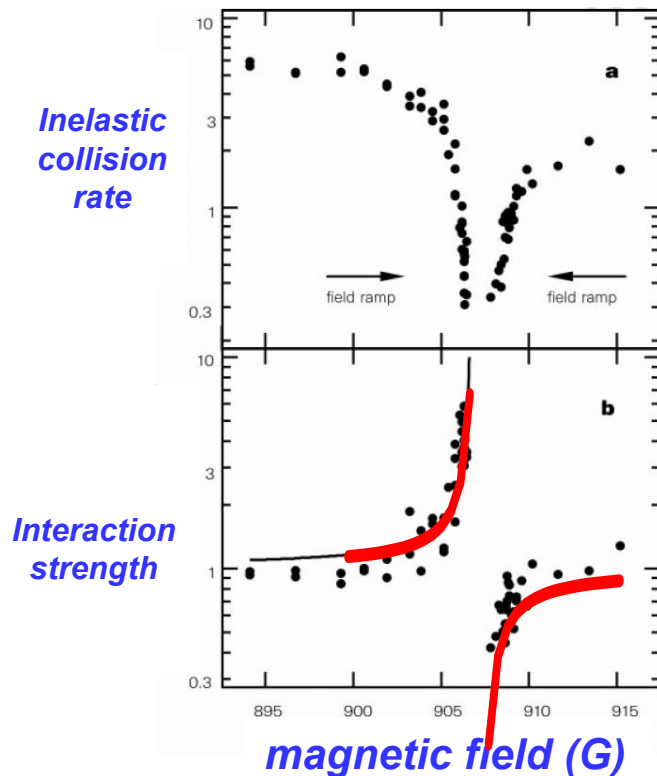
$$= \begin{pmatrix} \infty & 0 \\ 0 & 0 \end{pmatrix} \text{ for } R > a.$$

Result:
$$\frac{1}{a - r_0} = \frac{1}{a_{bg} - r_0} + \frac{\Gamma / 2r_0}{E_i}$$

A tunable Bose gas (98~01)

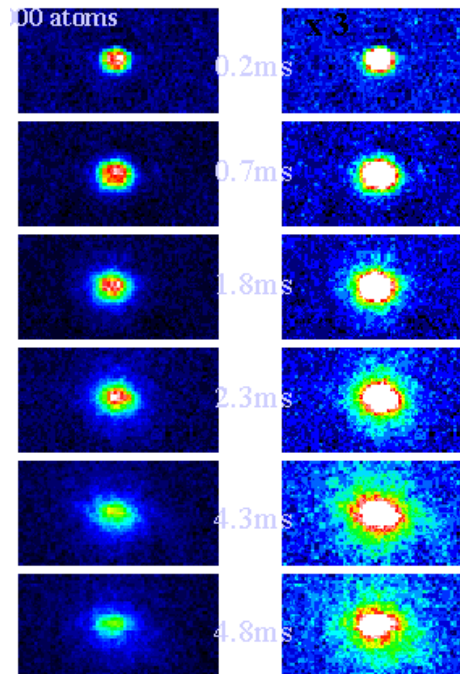
Feshbach resonance in Na

Ketterle group, 1997



BEC implosion and explosion (bosonova)

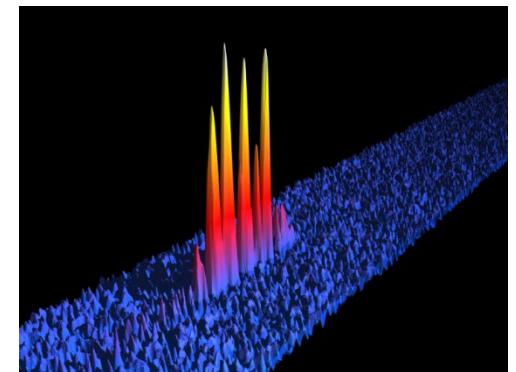
Wieman group, 2001



Bright Soliton

Hulet group, 2001

Salomon group, 2001

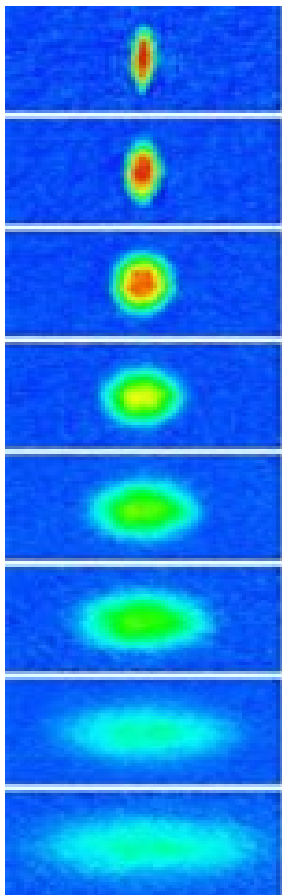


Feshbach resonances are found in all alkali species

Experiments on Fermionic condensates near a Feshbach resonance (02~05)

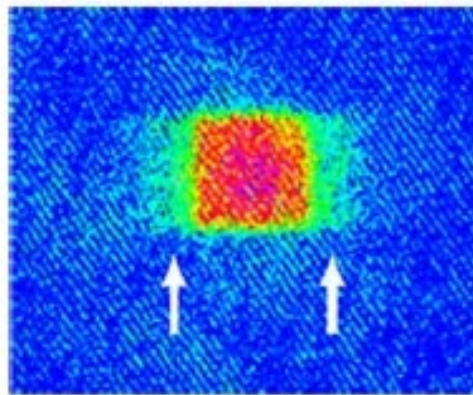
Hydrodynamic expansion

(Duke, ENS)



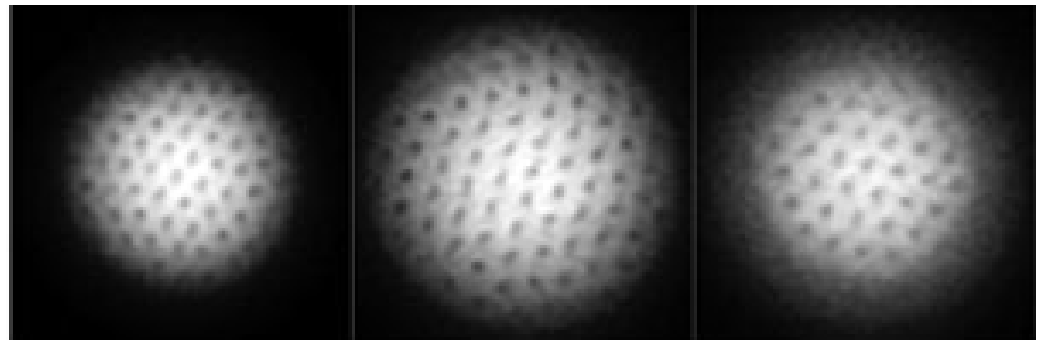
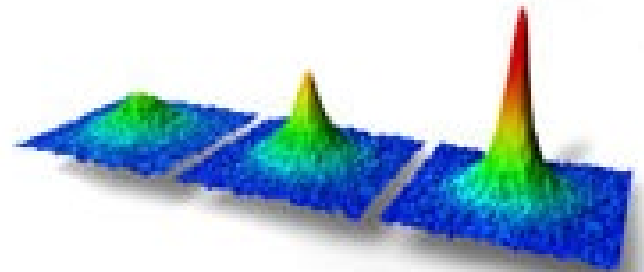
Band insulator

(ETH, Florence)

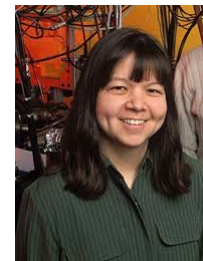
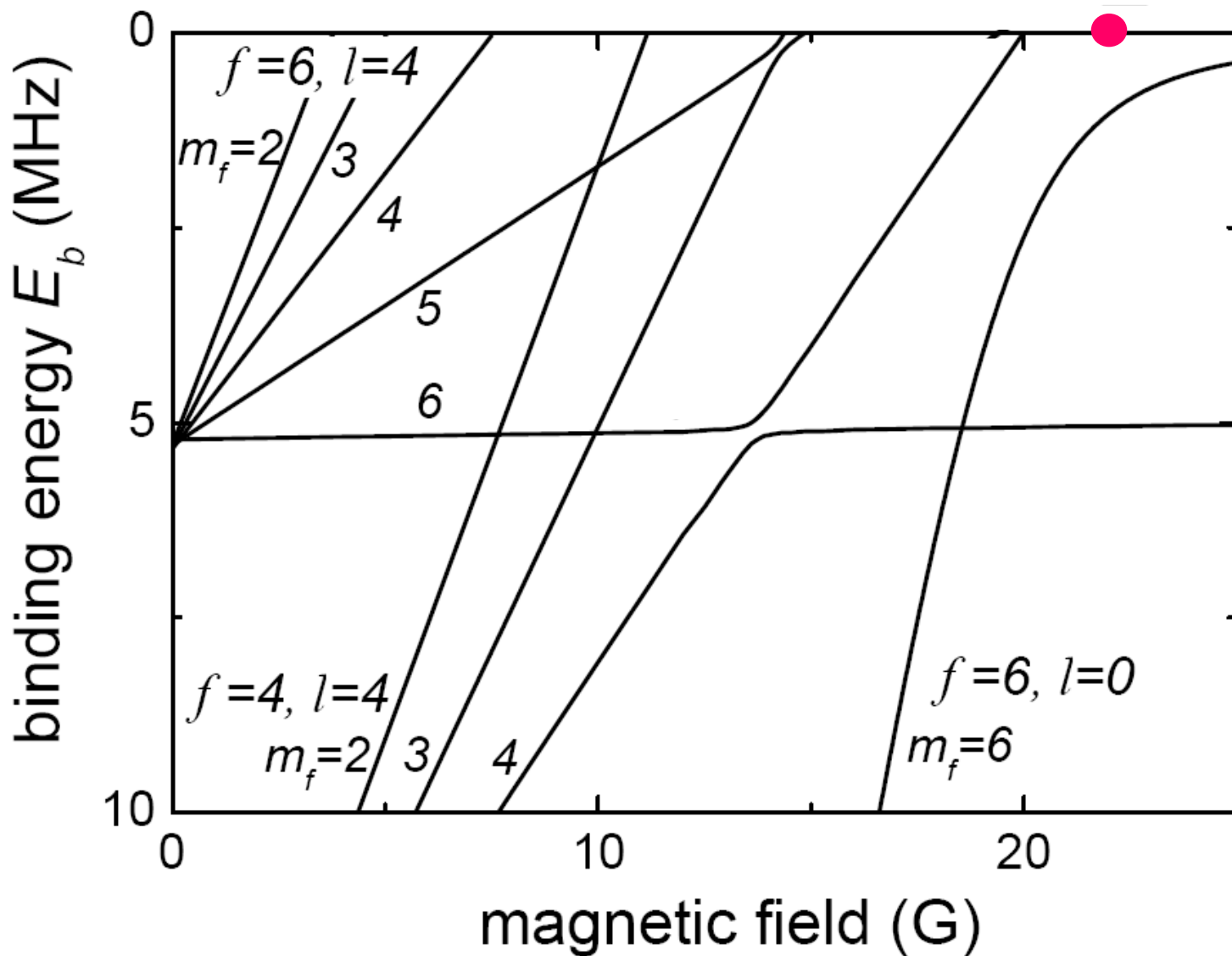


(BEC of Fermion pairs)

(Innsbruck, JILA, Rice, MIT, ENS)



Feshbach resonance: gateway to cold molecules (2003)



data
from
NIST
group

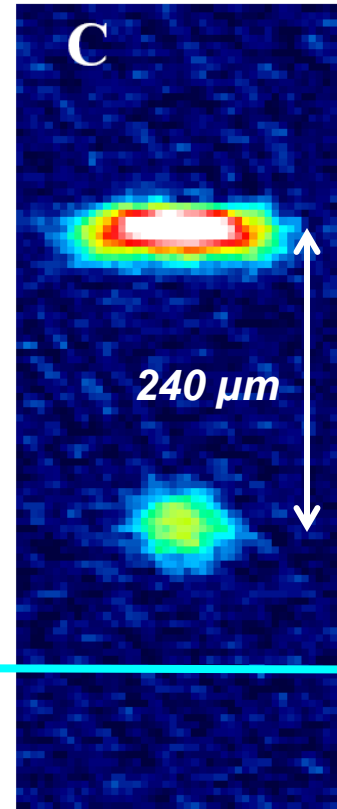
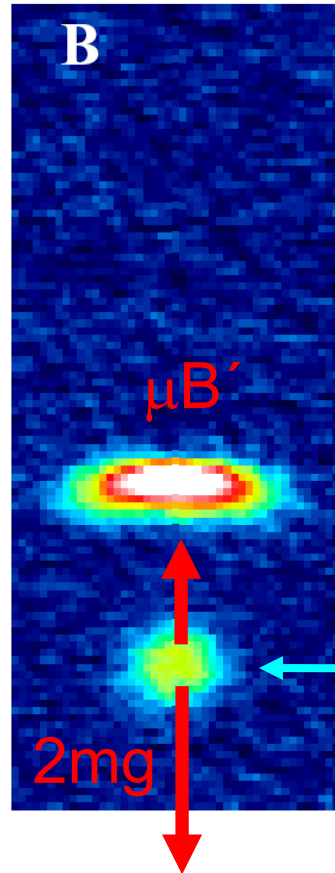
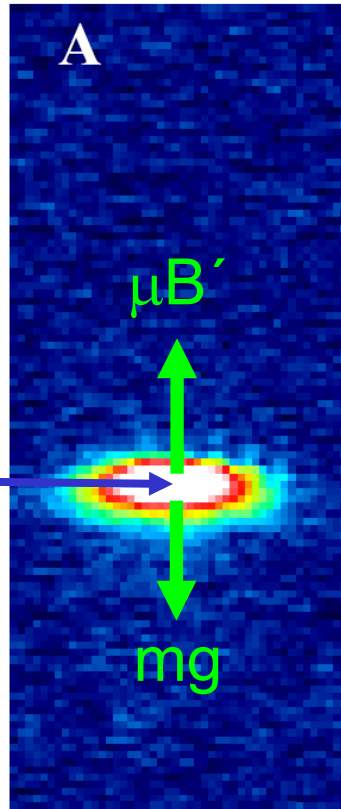
Observation of ultracold Feshbach molecules

$dB/dz=31.3$ G/cm

31.3 G/cm

50.3 G/cm

Atomic Cs BEC
60,000 atoms
 $\mu=0.755 \mu_B$

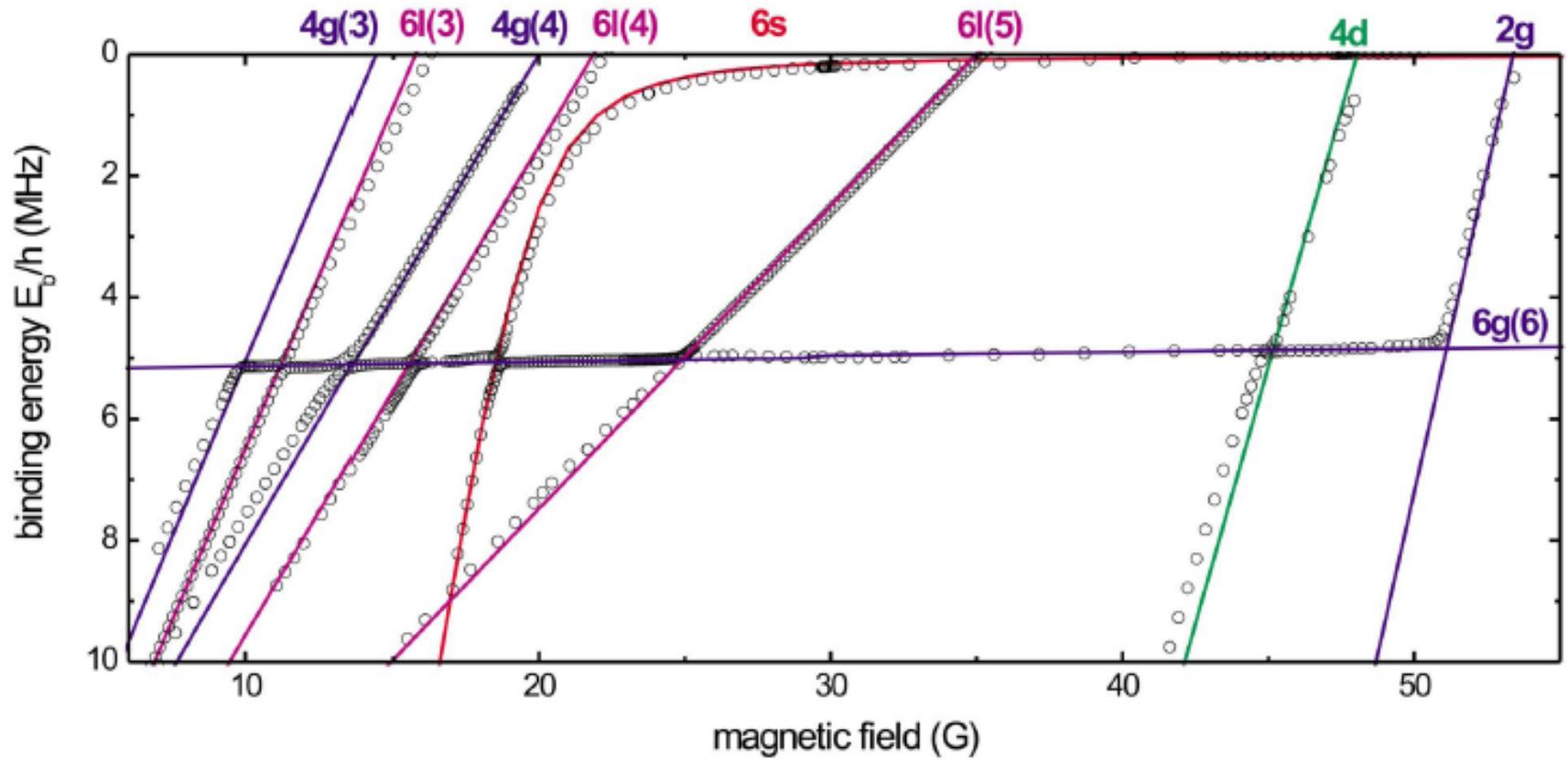


3,000 (10%)
Cs₂ molecules
 $\mu=0.95 \mu_B$
1.8 nK

J. Herbig et al., Science '03. Also see C. Regal et al., Nature '03 on K_2

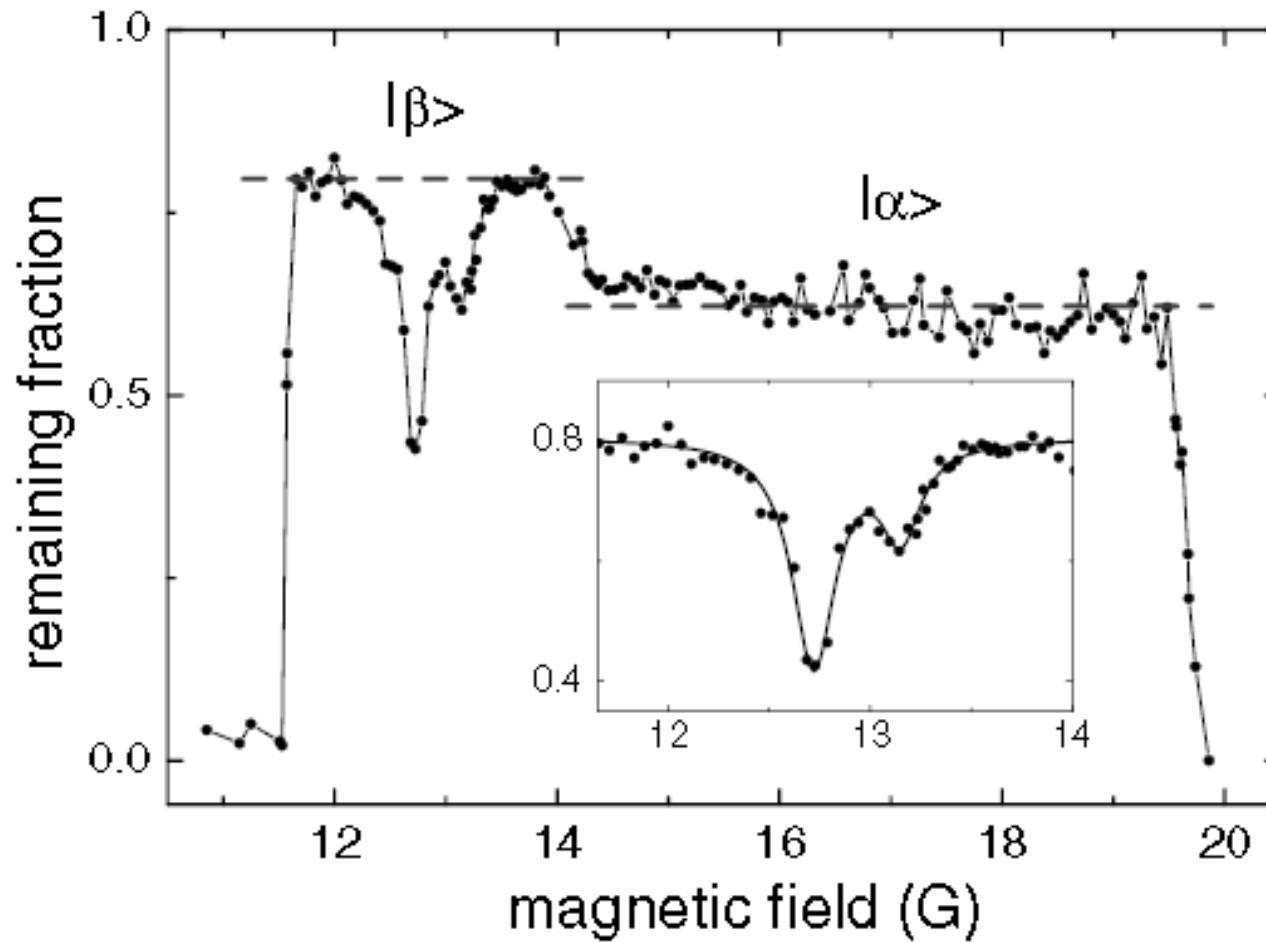
S. Durr et al., PRL '03 on Rb_2 , K. Xu et al., PRL '03 on Na_2 , S. Jochim et al., PRL '03 on Li_2

Map of Cs₂ molecular city



M. Mark et al., PRA 2007

New development of Feshbach resonances



Chin et al., PRL 2005

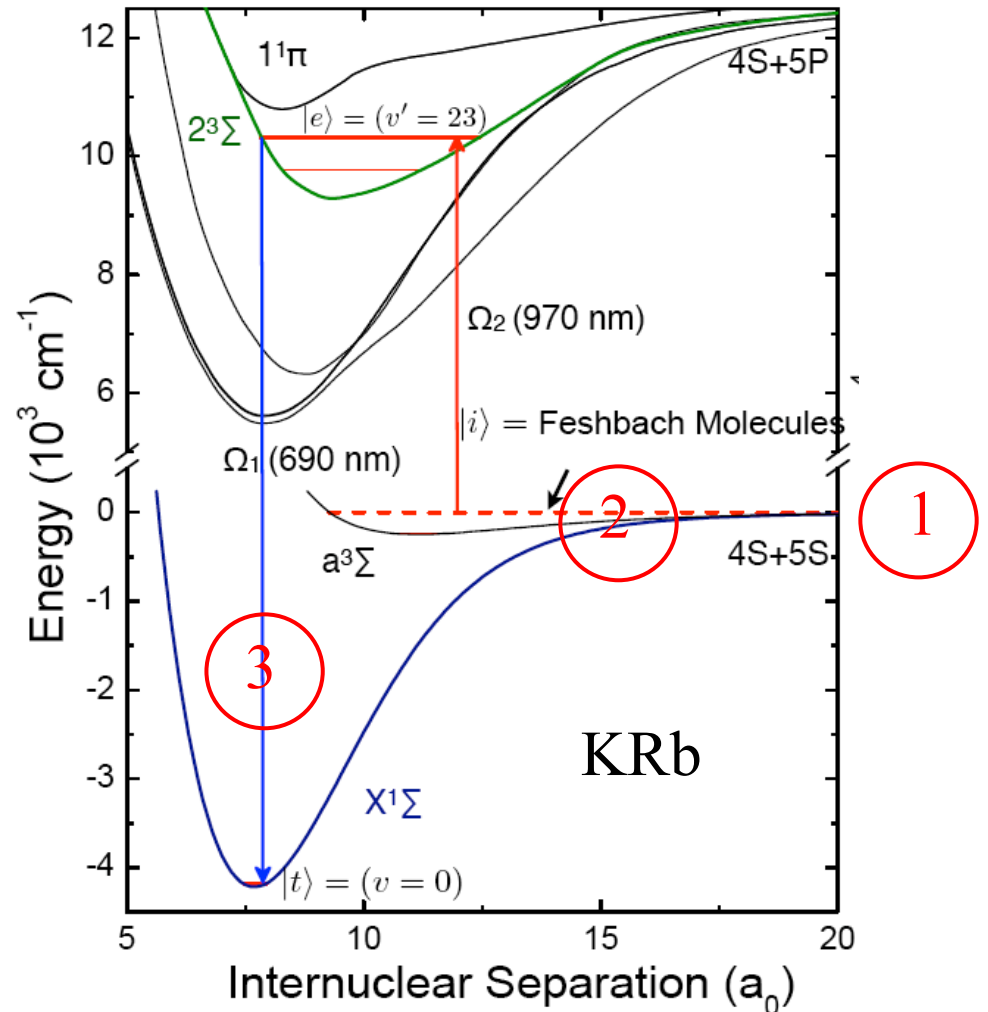
A High Phase-Space-Density Gas of Polar Molecules

SCIENCE VOL 322 10 OCTOBER 2008

K.-K. Ni,^{1*} S. Ospelkaus,^{1*} M. H. G. de Miranda,¹ A. Pe'er,¹ B. Neyenhuis,¹ J. J. Zirbel,¹
S. Kotochigova,² P. S. Julienne,³ D. S. Jin,^{1†} J. Ye^{1†}

40000 $^{40}\text{K}^{87}\text{Rb}$ molecules
 $v=0, J=0$, single spin level
 200 to 800 nK
 Density $\approx 10^{12} \text{ cm}^{-3}$

1. Prepare mixed atomic gas
2. Magneto-association to Feshbach molecule
3. Optically switch to $v=0$ ground state



Summary of Feshbach resonances

Tools to control atomic interactions

B field tunability: simulating condensed matter, nuclear physics,

Many-body applications: Solitons, BEC-BCS crossover, Hubbard model...

Pairing atoms into molecules: Feshbach molecules, Efimov trimers

Toward quantum manipulation: Coherent control of entanglement

New ideas

Test of fundamental constant variation

Bose fireworks, synthetic gauge field, quantum entanglement...

Thank you.