# Quantum logic gates

$|f\rangle = \prod_i U_i \, |i\rangle$

$|i\rangle$ encodes the $q$.
$|f\rangle$ reveals the $a$.
$U_i$ : unitary operation
$U_i = e^{-iH_i t/\hbar}$

Universal $q$ gates: A collection of gates that can approximate all possible unitary operations

$U = \prod_i U_i$

The most common choices are: $\left\{ \begin{array}{l} \text{single spin rot.} \\ \text{two spin control-NOT} \end{array} \right.$

There are infinitely many choices

Single qubit gates:

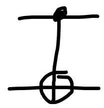
$-\boxed{X}- : \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad -\boxed{Y}- : \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. $\quad -\boxed{Z}- : \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$—— : \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad -\boxed{H}- : \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $\quad -\boxed{P_\theta}- : \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$

$$R_{\vec{n}}(\theta) = e^{-i\frac{\theta}{2}\vec{n}\cdot\vec{\sigma}} = \cos\frac{\theta}{2}\cdot\hat{1} - i\sin\frac{\theta}{2}(n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)$$

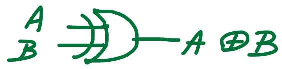
You got all above spin gates from $R_{\vec{n}}(\theta)$.

# Two qubit gate: Control-NOT $\approx$ XOR

CNOT  $= \begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 00 & 1 & 0 & 0 & 0 \\ 01 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & 0 & 1 \\ 11 & 0 & 0 & 1 & 0 \end{array}$

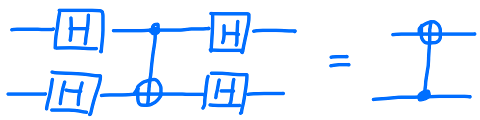
XOR

| | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |




Flip the 2nd spin only when the 1st one is 1.

1. It seems like CNOT does nothing to 1st bit. Wrong!

   Show that 


   2nd bit also controls 1st bit! This is quantum.

2. You may construct all Bell states with CNOT.

3. Equivalence of control phase gate


Proof: $I\otimes H \; CZ_{12} \; I\otimes H = I\otimes H \left[|0\rangle\langle 0|\otimes 1 + |1\rangle\langle 1|\otimes Z\right] I\otimes H$

$= |0\rangle\langle 0| H^2 + |1\rangle\langle 1| \otimes HZH$

$= |0\rangle\langle 0|\otimes 1 + |1\rangle\langle 1|\otimes X = CNOT_{12}$

Realization of CNOT gate

Ion trap: Mølmer-Sørensen gate.

Optical lattice: phase gate

tweezer array: Rydberg blockade