Now we will quantize both atomic energy and radiation field.

\[
\begin{align*}
\text{atom: } & \frac{\hbar}{2} \omega_0 a^\dagger a \\
\text{photons: } & \hbar \omega (a^\dagger a + \frac{1}{2}) \\
\text{atom-photon coupling: } & g \left( \omega_0 + \omega \right)(a^\dagger a) \\
\text{RWA: } & g \left( \omega_0 + \omega \right)(a^\dagger a) = g \left( \omega_0 + \omega \right)(a^\dagger a)
\end{align*}
\]

\text{Jaynes-Cummings model}

State \( |n\rangle = |0\rangle \otimes |n\rangle \quad \sigma = 0, 1, 2, \ldots \)

\text{model:}

\[
\begin{array}{c|c}
\hline
\text{no interaction:} & \text{assumed } \Delta = \omega - \omega_0 > 0 \\
\hline
\Delta & \hbar \omega_0 \\
\hline
\end{array}
\]

\[
\begin{align*}
|n+2\rangle & \quad |n+1\rangle \\
|n+1\rangle & \quad 1g, n+1\rangle \\
|n\rangle & \quad 1g, n\rangle \\
|n-1\rangle & \quad 1g, n-1\rangle \\
|n-2\rangle & \quad 1g, n-2\rangle \\
|n-3\rangle & \quad 1g, n-3\rangle \\
|n-4\rangle & \quad 1g, n-4\rangle \\
\hline
|n\rangle & \quad 1g, n\rangle \\
\end{align*}
\]

\text{RW coupling: } \sigma^+ a + \sigma^- a^\dagger \\
\text{CRW coupling: } \sigma a + \sigma^+ a^\dagger

\text{If we ignore CRW terms, all couplings occurs between } 1g, n+1\rangle \text{ and } 1g, n\rangle

\text{Consider only a pair of coupled states: } |n\rangle = (1g, n\rangle) \quad (1e, n\rangle)

\[
H = \frac{\hbar}{2} \omega_0 a^\dagger a + \hbar \omega (b^\dagger b + \frac{1}{2}) + g \sqrt{N} (\sigma^+ b + \sigma^- b^\dagger)
\]

\[
\langle H \rangle = \frac{\hbar}{2} \omega_0 \left( \sigma^+_1 \sigma^-_0 \right) + \hbar \omega \left( n + \frac{1}{2} \right) + g \sqrt{N} (\sigma^+_1 \sigma^-_0) = \frac{\hbar}{2} (a^\dagger a + a a^\dagger) + \text{odd } \sigma
\]

We have seen this a couple of times: Feshbach, Rabi flipping.

This is generally how \( 2 \) modes are coupled.
Thus the eigenstates are

\[ \begin{align*}
\Delta & \\
|n+1\rangle & \quad |n\rangle \\
\Delta & \\
|n\rangle & \quad |n-1\rangle \\
\end{align*} \]

Thus we have a new prediction for the light shift:

1. First, there are only eigenstates, \( 1g > D 1e \rangle \) are "dressed" by the radiation field. This is the dressed atom picture.

2. Light shift is the work done to bring an atom into the beam:

   \[ \text{Shift} = \frac{1}{2} (E - \Delta) \text{ for blue-detuned light } \Delta > 0 \]
   \[ = \frac{1}{2} (E + \Delta) \text{ for red-detuned light } \Delta < 0 \]

   \[ \Rightarrow \Delta E = \frac{1}{2} (\sqrt{\Delta^2 + g^2 n} - \Delta) \quad \text{for } \Delta \gg g \sqrt{n} \]

   \[ = \frac{1}{4} \frac{g^2 n}{\Delta} \quad \text{(compared to } \Delta E = \frac{F}{\Delta} \frac{T_x s}{\Delta T} \text{)} \]
Example I.

Can we include spontaneous emission?

\[ |l_e, n+1 \rangle \otimes |10 \rangle \rightarrow |l_e, n+1 \rangle \otimes |l_1 \rangle \]

\[ \rightarrow |l_e, n \rangle \rightarrow |l_e, n-1 \rangle \]

\[ |l_e, n \rangle \rightarrow |l_e, n-1 \rangle \]

\[ |l_e, n-1 \rangle \rightarrow |l_e, n-2 \rangle \]

\[ \ldots \rightarrow |g \rangle \]

Thus we predict spontaneously emitted photons should carry new components:

\[ g \rightarrow 0 \]

We emit\[ \text{strange new quarks appear}!! \]

Now question, illuminating some atoms with radiation at \( w_0 \), we actually get emission at \( w_0 - 2\Delta E_L = W_0 \) and \( w_0 + \Delta = W_0, 2W_0 - w \).

So if laser is detuned by \( \Delta = -1 \text{GHz} \), we see atoms emit photon at \( w_0 + 1 \text{GHz} = W_0 + 2 \text{GHz} \). We gain 2 GHz of energy!?

What's going on!? 

Mostly, the emitted photons have the same freq as the incident photons: Rayleigh scattering, (apart from light shift & recoil.)
Example 2: Autler–Townes effect

If laser is exactly on resonance

Many more interesting stuffs from dressed atom picture.