Quantum teleportation

Transfer a quantum state $\Psi$ from A to B. A and B are far separated.

Alice  $\Psi$  

Bob  $\Psi$  

no physical transport

Quantum state cannot be copied

no-cloning theorem

is this possible?

no-communication theorem

Proof of no-cloning theorem:

Proof of no-communication theorem:

$\rho_{AB} = \rho_A \otimes \rho_B$

$\rho_A = |A\rangle \langle A|$  

Alice does some operation:  $|A\rangle \rightarrow U_A |A\rangle \Rightarrow \rho_A = U_A^* |A\rangle \langle A| U_A$

$\rho_{AB} = U_A^* \rho_A U_A \otimes \rho_B$

$\rho_B = \text{tr}_A \rho_{AB} = \text{tr}(U_A^* \rho_A U_A) \otimes \rho_B = \text{tr}(\rho_A U_A U_A^*) \otimes \rho_B = \rho_B$

Nothing will happen to Bob.

This also implies no-cloning theorem. If Bob can duplicate any qubits, he would know whether a measurement has been made by Alice.

If Alice measures $|\uparrow\rangle$ Bob gets $|\downarrow\rangle$ $|\uparrow\rangle |\uparrow\rangle$ by duplication.

If Alice does not measure Bob gets random result.

So the best people have figured out to transfer a quantum state is

BB84 protocol: PRL 70, 1895 (1993)

Step 1. A and B share a pair of entangled spins $\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle_{AB} - |\downarrow\uparrow\rangle_{AB})$

Step 2. Alice projects her spin and the information into Bell state basis and collapse the wavefunction to one of them.
\[ |\psi^-angle = A (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) + \\
B (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) + \\
C (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) + \\
D (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

Say, \( |\psi^-\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle \) \( \Rightarrow \) LHS = \( \alpha |\uparrow\rangle + \beta |\downarrow\rangle - \alpha |\downarrow\rangle - \beta |\uparrow\rangle \)
\[ \Rightarrow \alpha = c + d, \quad -\beta = c - d \]
\[ \Rightarrow c = \frac{\alpha - \beta}{2}, \quad d = \frac{\alpha + \beta}{2} \]

Alice's measurement projects Bob's spin

- **Case A**: Bob gets \( [\sigma_z |\psi^-\rangle \]
- **Case B**: \( |\uparrow\rangle \]
- **Case C**: \( [\sigma_x |\psi^-\rangle \]
- **Case D**: \( [\sigma_y |\psi^-\rangle \]

Step 3: Alice tells Bob which case she got so Bob can determine \( |\psi^-\rangle \)

Remaining questions: How to perform a measurement in Bell state basis?

Is this the best we can do?

Teleport a continuous quantum variable \( Y(x) \)?

Let's think...

\[ x_2 - (x_1 - x_3) = x_1' - (x_1 - x_3) \approx x_1' \]
\[ P_1', P_2 + P_2' = P_1 + P_1 + P_2 \approx P_1' \]

No I offset my \( x_1 \) by \( (x_1 - x_3) \) and I offset my \( P_1 \) by \( -P_2 \) but then I am golden! \( (x_1 = x_1', P_2 = P_2') \)