Feshbach resonance

Energy relative to incident channel

\[ E \]

Bound state @ \( E_m \)

Closed channel \( \rightarrow \) incident channel (also an open channel)

Not accessible @ \( R \rightarrow \infty \) but can induce resonance

Open channel (accessible)

Resonance occurs when scattering energy \( E_s \) is near that of a bound state \( E_m \) in a closed channel.

A quantum optics analog:

\[ \begin{align*}
\text{incident wave} & \quad \downarrow \\
\text{bound state} & \quad \downarrow \\
\text{decay to lower open channel(s)} & \quad \downarrow
\end{align*} \]

Enhanced loss: inelastic Feshbach resonance

Enhanced elastic collisions: elastic Feshbach resonance

We will only discuss elastic Feshbach resonance here

You can write a paper to generalize what we do here to include inelastic resonance.

Model:

\[ \begin{align*}
E_m & \quad \downarrow \\
E & \quad \downarrow \\
E & \quad \downarrow \\
-\frac{\hbar^2}{2m} \nabla + V(r) & \quad \nabla \\
\hat{H} \Psi &= E \Psi \\
\Psi &= \begin{bmatrix} \psi_c(r) \\ \psi_0(r) \end{bmatrix} \\
V(r > r_0) &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
V(0 < r < r_0) &= \begin{bmatrix} E_c & V \\ V & E_0 \end{bmatrix}
\end{align*} \]
$E_c$: closed channel depth $\sim 1000K$
$E_o$: open channel depth $\sim 1000K$

$20\text{GHz} = 1K$

$V$: mixing between the two channels $\sim$ hyperfine $\sim 1K$

Scattering energy $E \sim k\hbar$

$E$: closed channel energy $\sim$ hyperfine $\sim 1K \to \infty$

$\Rightarrow$ outside the potential: $r\Psi(r>r_o) = (\sin(kr+a)) = \sin(kr)10>$

inside the potential: $r\Psi(r<r_o) = A_+\sin kr + 1+ + A_-\sin kr - 1->$

$1\pm >$ are the eigenstates.

$\begin{pmatrix}
1+>
1->
\end{pmatrix} =
\begin{pmatrix}
\cos\Theta & \sin\Theta
-\sin\Theta & \cos\Theta
\end{pmatrix}
\begin{pmatrix}
10>
1c>
\end{pmatrix}$

B.C. $\Psi(r_r) = \Psi(r_r+)$

$\Psi(r_r) = \Psi(r_r+)$

$\Rightarrow \Psi_c(r_r+) = \Psi_c(r_r-) = 0$

$\left|\frac{r\Psi}{r\Phi}\right|_{r_r} = k\cot(kr_0+\delta)$

$\left|\frac{r\Psi}{r\Phi}\right|_{r_r} = k_+\cot kr_0 \cos\Theta + k_-\cot kr_0 \sin\Theta$

$\Rightarrow k\cot(kr_0+\delta) = k_+\cos\Theta \cot kr_0 + k_-\sin\Theta \cot kr_0$

in the limit of small $\Theta$, and turn $\to 0$ so $1+ \approx 10$>

We recover single channel result (HN3)

$k\cot(kr_0+\delta) = k_+\cot kr_0$, thus we may rewrite
\[ k \cot (k \rho + \delta) = k \cot (k \rho + \delta_{bg}) + k - \delta \cot k \rho \]

In the limit \( k \to 0 \), we have

\[ \frac{1}{r_0 - a} = \frac{1}{r_0 - a_{bg}} + k - \delta \cot k r_0 \approx \frac{1}{r_0} \frac{T/2}{E_m} \]

\[ \Rightarrow a = a_{bg} - \frac{r_0 T'}{E_m + \Delta E} \]

Coupling strength

Self energy of the med.

To show the magnetic tunability, we assume \( E_m \) can be tuned as \( E_m(B) = E_m + \mu m B \)

Show that (HW3) we arrive at \( a = a_{bg} \left( 1 - \frac{\Delta B}{B - B_0} \right) \)