2018P238 Modern Atomic Physics 2018 Name: ___________

Home Work 1

(Due: 10/23/2018)

1. In this problem we study the time evolution of a spin-1/2 atom in a combination of static and rotating fields $\vec{B}(t) = -(B_1 \cos\omega t, B_1 \sin\omega t, B_0)$ . The total Hamiltonian is

$$H = -\vec{\mu}\cdot\vec{B} = \frac{\hbar}{2}\begin{pmatrix} \omega_0 & \omega_R e^{-i\omega t} \\ \omega_R e^{i\omega t} & -\omega_0 \end{pmatrix},$$

where $\vec{\mu}$ is the atomic magnetic moment, $\omega_0=\gamma B_0$ and $\omega_R=\gamma B_1$ are the Larmor and Rabi frequency associated with the static field $B_0$ and and rotating field $B_1$, respectively, and $\gamma$ is the gyromagnetic ratio.

The wavefunction basis is $\begin{pmatrix}1\\0\end{pmatrix} \equiv |e>$, $\begin{pmatrix}0\\1\end{pmatrix} \equiv |g>$ and the time evolution of the state $|\psi(t)>= a_g(t)|g> + a_e(t)|e>$ is determined by the two complex coefficients $a_g(t)$ and $a_e(t)$.

a) Find the equations of motion for the system, i.e. derive the differential equations for $a_g(t)$ and $a_e(t)$.

b) Solve the equations of motion and find $a_g(t)$ and $a_e(t)$ in terms of $a_g(0)$ and $a_e(0)$.

c) Given the initial condition $a_g(0)=1$ and $a_e(0)=0$, show that the probability to find the system in the excited state follows the Rabi's formula:

$$|a_e(t)|^2 = \frac{\omega_R^2}{\omega_R^2 + (\omega-\omega_0)^2}\sin^2\frac{\sqrt{\omega_R^2+(\omega-\omega_0)^2}\,t}{2}$$

2. Rabi flopping as quantum operations
   A quantum operation on an atom can be implemented by a magnetic field pulse described in 1., which converts an initial quantum state $|\psi_i>$ into a final state $|\psi_f>=\hat{U}|\psi_i>$, where $\hat{U}$ is a 2x2 matrix.

   a) Determine $\hat{U}_{\pi/2}$ for a resonant $(\omega=\omega_0)$ magnetic field pulse of duration $t=\frac{\pi}{2\omega_R}$.

   b) Show that $\hat{U}_{\pi/2}$ can serve as a 50% - 50% beam splitter if the atom is initially prepared in the ground state $\begin{pmatrix}0\\1\end{pmatrix}$.

   (Hint: writing $\begin{pmatrix}a_e\\a_g\end{pmatrix}=\hat{U}_{\pi/2}\begin{pmatrix}0\\1\end{pmatrix}$, you can show that $|a_e|^2=|a_g|^2=\frac{1}{2}$. Calculation would be easier if you do it in the rotating frame)

3. Atom clock operates based on the following recipe (Ramsey spectroscopy)

   Step 1: prepare an atom in the ground state $\begin{pmatrix}0\\1\end{pmatrix}$

   Step 2: apply a π/2 pulse $\hat{U}_{\pi/2}$

   Step 3: wait for some free evolution time T: $\hat{U}(\omega_R=0,t=T)$

   Step 4: apply another π/2 pulse $\hat{U}_{\pi/2}$

   Show that after all steps, the populations in the excited state is given by $|c_e(T)|^2=\cos^2\frac{\omega_0 T}{2}$.