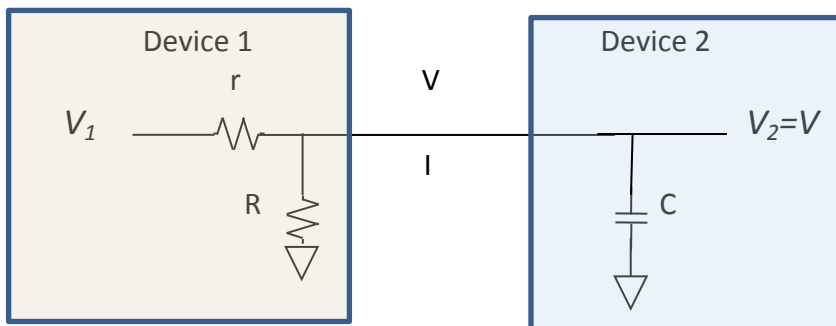


1. Impedance: resistors, capacitors, and inductors

(A) Device 1 delivers a signal to Device 2. Calculate the output impedance and input impedance as a function of the angular frequency ω .

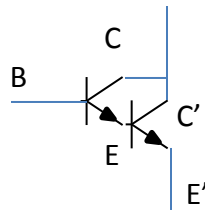
(Hint: you may assume the actual output voltage and current are V and I . Thus output impedance is $Z_{\text{out}} = |dV/dI|$ expressed in terms of r and R . Input impedance is $Z_{\text{in}} = |dV/dI|$ expressed in terms of C .)



(B) Bandpass. Insert an RLC circuit between Device 1 and Device 2 such that only the signal at a desired angular frequency ω_0 can be delivered to Device 2 with 100% efficiency. Frequencies far from ω_0 will be suppressed.

2. Transistors and amplifier

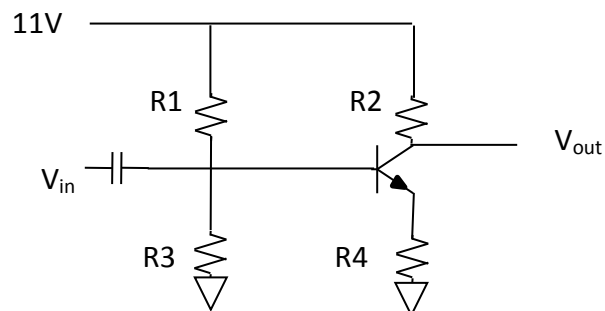
(A) Darlington pair:



The above Darlington pair is made of 2 identical diodes with current amplification of $I_C = \beta I_B$ and $\beta=50$ is the amplification factor. Given a small base current $I_B = 1 \mu\text{A}$, determine the currents flowing through the collector terminals of each transistor I_C and $I_{C'}$.

(B) Transistor amplifier

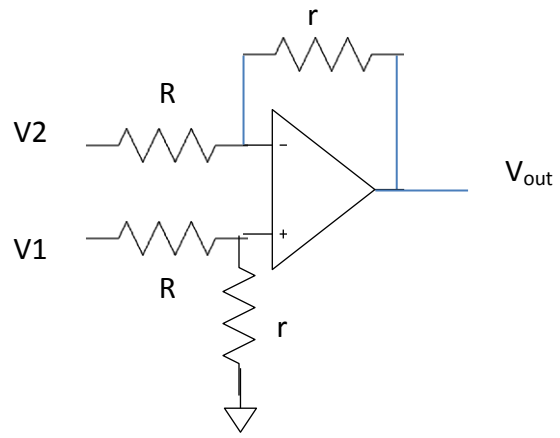
You wish to amplify an AC signal V_{in} by a factor of 20. How would you choose R_1 , R_2 , R_3 and R_4 to accomplish this task.



(c) Continue (B), derive V_{out} as a function of V_{in} when the amplification is working properly.

3. Operational amplifiers

(A) Calculate the output V_{out} of the following circuit, and explain its function.



(B) Design an op-amp circuit to process two signals V_1 and V_2 such that $V_{out}(t) = V_1(t) + 2dV_2(t)/dt$.

4. Feedback and car driving

You are driving on the highway at a constant speed, and your car drifts sideways from the center of the lane with an excursion x . Your goal is to keep the car centered or $x=0$. To reach this goal, you turn the steering wheel by θ .

For small excursion x , the transverse motion of the car can be described by

$$\frac{d^2x}{dt^2} = \alpha\theta + f(t),$$

where $f(t)$ is the random external force pushing the car sideways due to imperfect road condition, α is the steering sensitivity.

(A) Write the equation in the frequency domain.

(Hint: Use Fourier transform $y(t) = \int y(\omega)e^{-i\omega t} dt$, where $y(t)$ is a function in time domain, and $y(\omega)$ is the function in the frequency domain.)

(B) If the external force is a white noise $f(\omega) = f_0 = \text{constant}$. Show that your car is very easily influenced by low frequency components of the external force if you do not control the steering wheel $\theta=0$.

(C) One way to control your car is to turn the steering wheel in the opposite direction of the excursion $\theta = -Gx$, with a constant proportional gain $G=\text{constant}$. Such approach seems reasonable, but leads to failure. Show that your car will still lose stability.

(D) Come up with a feedback strategy $\theta(\omega) = G(\omega)x(\omega)$ with a simple gain function $G(\omega)$ such that you can keep the car more stable at all frequencies $\omega \geq 0$.

(Hint: a car is considered more stable if the excursion x does not diverge at any frequency, and is smaller than that of an unsteered car at all frequencies.)