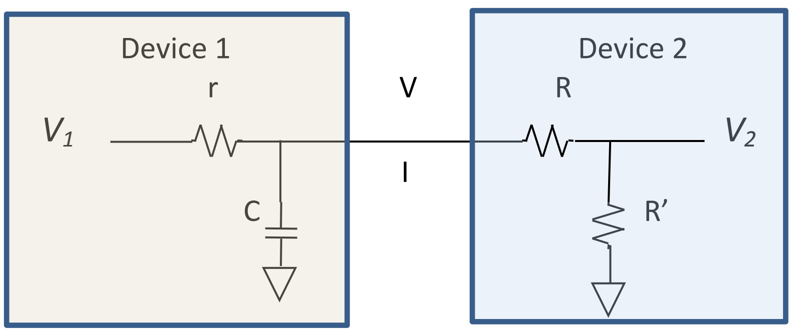
PHY226 Electronics **Midterm (5/7/2019)** **Name: ________________**

1. Impedance: resistors, capacitors, and inductors

(A) Device 1 delivers a signal to Device 2. Calculate the output impedance and input impedance as a function of the angular frequency *w*.
(Hint: you may assume the actual output voltage and current are V and I. Thus output impedance is Z_{out} = |dV/dI|, expressed in terms of r and C. Input impedance is Z_{in} =|dV/dI| expressed in terms of R and R'.)



Solution:

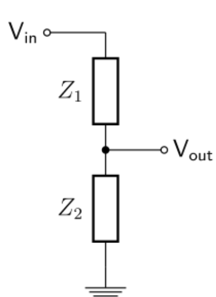
$$I = \frac{V_1 - V}{r} - \frac{V}{Z_c} \Rightarrow dI = -\left(\frac{1}{r} + \frac{1}{Z_c}\right) dV \Rightarrow Z_{out} = \left|\frac{dV}{dI}\right| = \left|\frac{rZ_c}{r + Z_c}\right| = \frac{r}{\sqrt{1 + \omega^2 r^2 C^2}}$$

$$V = I(R + R') \Rightarrow Z_{in} = \left|\frac{dV}{dI}\right| = R + R'$$

(B) Bandbloch: Design an *RLC* circuit between Device 1 and Device 2 such that only signal at a desired angular frequency w_0 will be completed blocked.

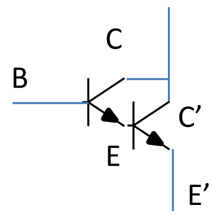
Solution: $V_{out} = \frac{Z_2}{Z_1 + Z_2} V_{in}$. The simplest way to satisfy the required condition is Z_1=R

And $Z_2(\omega_0) = iw_0 L + \frac{1}{i\omega_0 C} = 0$. Thus we can choose $LC = \frac{1}{\omega_0^2}$.



2. Transistors and amplifier

(A) Darlington pair:

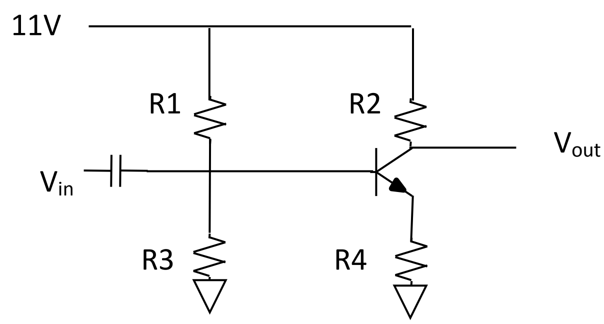


The above Darlington pair is made of 2 identical diodes with current amplification of $I_c = \beta I_B$ and β=*60* is the amplification factor. Given a small base current I_B = 1 μA, determine the currents flowing through the collector terminals of each transistor I_C and $I_{C'}$.

Solution: β μA, $\beta(\beta+1)$ μA

(B) Transistor amplifier

You wish to amplify an AC signal V_{in} by a factor of 10. How would you choose R1, R2, R3 and R4 to accomplish this task.



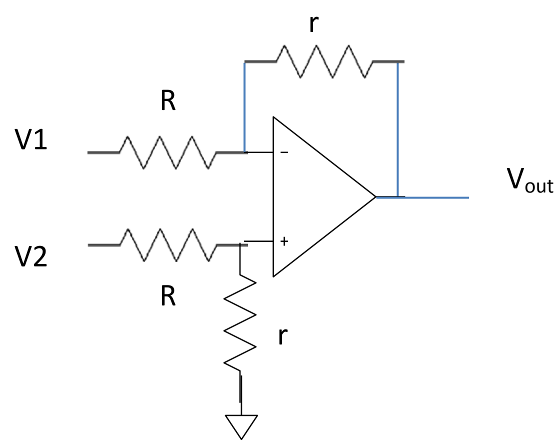
Solution: R2/R3=10, 1.6 > 11 R3/(R1+R3) > 0.6.

(c) What is the maximum range of the input voltage V_{in} that the amplifier circuit works?

Solution: V_{out} can reach maximum of 11 V (transistor fully off) and minimum of 1V (transistor fully on) with an amplification of 10. Thus V_in can swing no more than 1V and thus $-0.5V < V_{in} < 0.5V$. (This happens when the R1 and R3 properly bias the input to (0.6+1.6)/2 = 1.1 V.)

3. Operational amplifiers

(A) Calculate the gain $G=V_{out}/V_{in}$ of the following circuit, and explain its function.



Solution: Difference amplifier Vout = (V2-V1)(R/r).

(B) Design an Op-amp circuit to process two signals V1 and V2 such that $V_{out}(t)=dV_1(t)/dt+2V_2(t)$.

Solution: We can write the desired formula as $V_{out} = 2\left(V_2 - \frac{1}{2}\frac{dV_1}{dt}\right)$, which in frequency space is $V_{out} = 2\left(V_2 - \frac{i\omega}{2}V_1\right)$. Thus we can combine a differentiator on V1 with a gain of $G = \frac{i\omega}{2}$ together with a difference amplifier as shown in (A) with a gain of R/r=2.

4. Feedback and car driving

You are driving on the highway at a constant speed, and your car drifts sideways from the center of the lane with an excursion *x*. Your goal is to keep the car centered or *x=0*. To reach this goal, you turn the steering wheel by θ.

For small excursion *x*, the transverse motion of the car can be described by

$$\frac{d^2x}{dt^2} = \alpha\theta + f(t),$$

where *f(t)* is the random external force pushing the car sideways due to imperfect road condition, α is the steering sensitivity.

(A) Write the equation in the frequency domain.
(Hint: Use Fourier transform $y(t) = \int y(\omega)e^{-i\omega t}d\omega$, where *y(t)* is the function in time domain, and *y(ω)* is the function in the frequency domain.)

Solution: $-\omega^2 x(\omega) = \alpha\theta(\omega) + f(\omega)$

(B) If the external force is a white noise $f(w) = f_0$ = constant. Show that your car is very easily influenced by low frequency components of the external force if you do not control the steering wheel $\theta=0$.

Solution: We have $(\omega) = -\frac{f_0}{\omega^2 - G}$. When G=0, x diverges at DC $\omega = 0$.

(C) One way to control your car is to turn the steering wheel in the opposite direction of the excursion $\theta = -Gx$, with a constant positive gain *G>0*. Such approach offers negative feedback, but shows that it still leads to diverging instability.

Solution: Given G=constant, x diverges at $\omega = \sqrt{G}$,

(D) Come up with a feedback strategy $\theta(\omega) = G(\omega)x(\omega)$ with a simple gain function $G(\omega)$ such that you can keep the car more stable at all frequencies $\omega \geq 0$.
(Hint: a car is considered more stable if the excursion x does not diverge at any frequency, and is smaller than that of an unsteered car at all frequencies.)

Solution: One simplest solution is to employ an integration gain $G = \frac{1}{i\omega C}$, which gives $x = -\frac{f_0}{\omega^2 + \frac{i}{\omega C}}$. The oscillation amplitude $|x| = \frac{f_0^2\omega C}{\sqrt{1+C^2\omega^6}}$ does not diverge at any frequency.