Lecture 15-1  Heat engine

1st see the video on how a modern ICE works:
Internal Combustion Engine (ICE)

Intake Stroke
Compression Stroke
Ignition Stroke
Power Stroke
Exhaust Stroke

In this lecture we will understand the heat engine

Based on simple models: ideal gas

Background knowledge & approximation

Approximate air as ideal gas

Energy of the gas: \( U = N \langle N \rangle K T \)

\( \gamma = \frac{5}{3} \) for atom

\( \approx \frac{5}{2} \) for air

\( \frac{1}{2} k T = \frac{1}{2} m v^2 \)
Isochoric process \((V=\text{const.})\)

Specific heat: heat needed to raise unit mass by \(1^\circ\)

\[
C_v = \frac{1}{M} \frac{dQ}{dT} = \frac{1}{M} \frac{dU}{dT} = \frac{N}{M} \gamma k = \frac{1}{m} \gamma k
\]

mol. mass.

Under the assumption the volume does not change \(\Delta V=0\).

\[
\Rightarrow C_v = \gamma \frac{k}{m} \text{ is called isochoric specific heat. } C_v
\]

\(V=\text{const}\)

If pressure is held constant \((p=\text{const})\)

\[
\Delta U = \Delta Q - P \Delta V = \Delta Q - P A \Delta x
\]

\[
P = 1 \text{ atm } = \text{ const}
\]

\[
\Delta U = \Delta Q - P A \Delta x = \frac{1}{M} \frac{dQ}{dT} = \frac{1}{M} \left( \frac{dU}{dT} + P \frac{dV}{dT} \right)
\]

\[
C_p = \frac{1}{M} \frac{dQ}{dT} = \frac{1}{M} \left( \frac{dU}{dT} + P \frac{dV}{dT} \right)
\]

We need more energy

\[
C_p : C_v = \gamma + 1 : \gamma = \frac{N}{M} \gamma k + \frac{N}{M} \gamma\frac{k}{M} = \frac{N}{M} \gamma + 1 + \frac{N}{M} \gamma = \frac{N}{M} \gamma + 1
\]

\[
\Rightarrow C_p = \gamma \frac{1}{\gamma} C_v \text{ is isobaric specific heat. }
\]

adiabatic process

The third process is for an isolated system \((\Delta Q=0)\)

\[
U=\gamma NkT
\]

\[
\Delta U = -P \Delta V = \gamma Nk \Delta T = \gamma \Delta P V
\]

\[
\Rightarrow P \Delta V + \gamma P \Delta V + \gamma V \Delta P = 0
\]

\[
\Rightarrow (\gamma + 1) P \Delta V + \gamma V \Delta P = 0
\]

\[
\Rightarrow (\gamma + 1) \frac{\Delta V}{V} + \gamma \frac{\Delta P}{P} = 0 \Rightarrow \frac{\Delta P}{P} = \text{ const.} \Rightarrow PV^{\gamma+1} = \text{ const}
\]

Lastly, if \(T=\text{const}\), \(PV=\text{const}\). \(\Delta U=0\). Isothermal process

\[
\Delta U = \Delta Q - P \Delta V = 0 \Rightarrow \Delta Q = P \Delta V
\]

if you expand a gas, it \(T\) absorbs heat from environment.
(1) $\Delta V = 0$

(2) $P=0$

(3) adiabatic

(4) $T_{\text{H}_2\text{O}} = T_{\text{room}}$
Heat absorbed from the hot source: \( \Delta Q_H = \int_A^B P \, dV = \boxed{AB} \)

Heat released to the cold plate: \( \Delta Q_C = \int_C^P P \, dV = \boxed{PC} \)

Energy conversion efficiency: \( \frac{\Delta \text{Work}}{\Delta Q_H} = \frac{\text{ABCD area}}{\text{AB area}} \)

\[ \text{AB area} = \int_A^B P \, dV = NKTH \ln \frac{V_B}{V_A} = NKTH \ln \frac{V_B}{V_A} \]

\[ \text{BC area} = \int_B^C P \, dV = C_1 \int_B^C V^{-\alpha} \, dV = C_1 \frac{1}{-\alpha+1} (V_C^{-\alpha+1} - V_B^{-\alpha+1}) \]

Use \( C_1 = P_B V_B^\alpha = P_C V_C^\alpha \), \( C_2 = P_A V_A^\alpha = P_B V_B^\alpha \)

\[ P_A V_A = P_B V_B = NKTH \], \[ P_B V_B = P_C V_C = NKTC \]

Show that:
1. \( \frac{V_C}{V_B} = \frac{V_B}{V_A} \)
2. DC area = \( NKTC \ln \frac{V_C}{V_B} \)
3. BC area = AD area
4. ABCD area = \( NK(T_H-T_C) \ln \frac{V_B}{V_A} \) \( \propto T_H - T_C \)
5. \( \Delta Q_H = \text{AB area} = NKTH \ln \frac{V_B}{V_A} \propto TH \) \( T_H > T_C \)

\( \Rightarrow \) Carnot cycle efficiency: \( \frac{\text{ABCD area}}{\text{AB area}} = \boxed{1 - \frac{T_C}{T_H}} \)

is known to be the best efficiency for a thermal engine exchanging heat with 2 reservoirs at \( T_H \) and \( T_C \). Unless \( T_C = 0 \), we never reach 100%.

\( 1 - \frac{300}{900} \)
Lecture 15-2  Heat engines

Simplest and illustrative example of motors driven by heat.

\[ T = T_H \quad \rightarrow \quad T_H \rightarrow T_c \quad \rightarrow \quad T_c \quad \rightarrow \quad T_c \rightarrow T_H \]

**Step 1:** Isothermal expansion

- \( \Delta Q > 0 \) absorb heat
- \( PAV > 0 \) does work
- \( \Delta U = \Delta Q - PAV = 0 \)
- \( PV = NkT_H = \text{const.} \)

**Step 2:** Adiabatic expansion

- \( \Delta Q = 0 \)
- \( PAV > 0 \)
- \( \Delta U < 0 \)
- \( PV^{\alpha} = C_1 \)

**Isothermal compression**

- \( \Delta Q < 0 \) cool down
- \( PAV < 0 \) negative work
- \( \Delta U = \Delta Q - PAV = 0 \)
- \( PV = \text{const.} \)

**Adiabatic compression**

- \( \Delta Q = 0 \)
- \( PAV < 0 \)
- \( \Delta U = -PAV \)
- \( PV^{\alpha} = C_2 \)

How much work does a Carnot engine do per cycle?

\[ PV = NkT_H \quad \rightarrow \quad PV = NkT_c \]

Work done by the system:

\[ \int_A^B + \int_C^D PdV = \text{area} \]
Typical ICEs use $T_h = 600\, ^\circ C$; Carnot efficiency $= 55\%$. Our cars only reach $35\%$ (Otto cycle), wasting $25\%$ of the energy into heat.

Otto cycle: generates more energy, but even less efficient than Carnot cycle.