Physics 143b: Honors Waves, Optics, and Thermo

Spring Quarter 2020

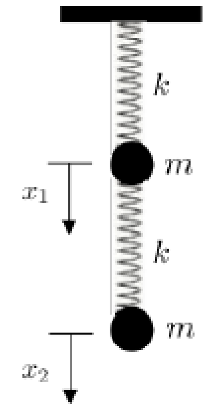
Problem Set #2

Due: 11:59 pm, Thursday, April 15. Please submit to Canvas.

1. **Math** Determine the eigenvalues and eigenvectors of the following matrices (5 points each)
   (a) $\begin{pmatrix} A & -A \\ A & A \end{pmatrix}$
   (b) $\begin{pmatrix} A & -A & A \\ -A & 0 & -A \\ A & -A & A \end{pmatrix}$
   (c) A particle of mass 1 is moving on a two dimensional plane with potential energy $V(x,y) = x^2 + y^2 - xy - 3x$. Determine the equilibrium position (x_0, y_0), where the potential energy is at the minimum, and the two eigen-frequencies of the particle moving near the potential minimum.
   (Hint: You may define a new coordinate $(u = x - x_0, v = y - y_0)$. Then show that in the new frame $F = ma$ gives
   $$u'' = -2u - v$$
   $$v'' = -2v - u.$$
   You can then derive the eigenfrequencies from the equation.)
   (d) Derive or simply argue what eigen-frequencies are if the particle is moving near the minimum of the potential $V(x,y) = e^{x^2+y^2-xy}$.

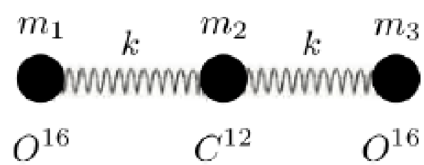
2. **Damping of coupled oscillators** (10 points each)
   What will happen if you couple two damped oscillators? In HW1 problem 1 we solved $x'' + 4x' + 3x = 0$, which describes an overdamped oscillator. Now we couple two of them as
   $$x'' + 4x' + 3x = 2y$$
   $$y'' + 4y' + 3y = 2x$$
   (a) Write the equations in the vector-matrix form as $\vec{x}''(t) + \hat{\gamma}\vec{x}'(t) + \widehat{M}\vec{x}(t) = 0$, where $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ and determine the matrices $\hat{\gamma}$ *and* $\widehat{M}$.
   (b) Determine the four eigen-frequencies and show that some modes apparently become underdamped.
   (Hint: a complex and not pure imaginary root corresponds to underdamped motion.)
   (c) Now consider two identical, critical damped oscillators coupled to each other, namely,
   $$x'' + 4x' + 4x = \epsilon y$$
   $$y'' + 4y' + 4y = \epsilon x.$$
   Show that 2 modes are overdamped and 2 modes are underdamped with oscillation frequency given by $\sqrt{\epsilon}$.

3. **Two masses on two vertical springs** (10 points each)
Two identical masses are attached to two massless springs as shown. Considering only motion in the vertical direction, solve for the motion of each mass about their equilibrium positions.



(a) Given the gravitational pull of *mg* and assuming no friction, write down the differential equations that describe the motion of the masses in terms of the deviations of their positions x_1 and x_2 from the equilibrium position, see Figure. Show that gravitational pull does not show up in the equations.
(b) Determine and describe the eigenmodes and their frequencies. (Hint: you may assume $\omega_0 = \sqrt{k/m}$.)
(c) At $t=0$, the lower mass receives a kick with velocity $x_2'(0)=v_2$, while $x_1(0)= x_1'(0)= x_2(0)$, determine their subsequent motion $x_1(t)$ and $x_2(t)$.

4. **Vibrations of a CO2 molecule (10 points each)**
A CO2 molecule can be modeled as a central carbon atom with mass m_2 connected by 2 identical springs with spring constant k to two oxygen atoms with mass $m_1 = m_3$.



(A) First we consider linear motion of the 3 atoms along the symmetry axis. Write down the differential equations that describe their motion x_1, x_2 and x_3 and determine the eigenfrequencies.
(B) Describe the eigenmodes. In principle there should be 3 eigenmodes for 3 oscillators. Explain why you only get two normal modes? Where is the third mode?