# Physics 143b: Honors Waves, Optics, and Thermo

Spring Quarter 2021

Final

Due: 12:00 am, Wednesday, June 2. Please submit to Canvas.

**Problem 1 Please circle your choice (there could be 0, 1 or more than 1 answer.) (4 points each)**

1.1 A harmonic oscillator described by $x''(t) + \omega_0^2 x(t) = f \cos \omega t$ is initially driven at frequency $\omega$ with amplitude $f = 1$. At $t = 0$, we stop driving it, $f = 0$. What is the frequency of the subsequent motion?

1. $\omega$ 2. $\omega_0$ 3. both $\omega$ and $\omega_0$

1.2 $CO_2$ is a linear molecule O-C-O. How many vibrational eigenmodes does it have for small amplitude oscillations along the molecular axis?

1. 1 2. 2 3. 3 4. 6

1.3 What are the functions that can satisfy the wave equation $\partial_t^2 \psi(x,t) = v^2 \partial_x^2 \psi(x,t)$?

1. $\psi_1 = e^{x+vt} \sin(x - vt)$ 2. $\psi_2 = \sin x \sin vt$ 3. $\psi_3 = 2^x 2^{vt}$ 4. $\psi_4 = \sin x + v \cos t$

1.4 $\vec{A} = (A_x, A_y, A_z)$ is a vector field. Show that $\vec{\nabla}(\vec{A} \cdot \vec{A})$ equals to

1. $2(\vec{\nabla} \cdot \vec{A})\vec{A}$ 2. $2(\vec{A} \cdot \vec{\nabla})\vec{A}$ 3. $2(A_x \partial_x A_x + A_y \partial_y A_y + A_z \partial_z A_z)$ 4. $2\vec{A}$

1.5 What is the Fourier series expansion of the function $f(-\pi < x < \pi) = |x|$ and $f(x + 2\pi) = f(x)$? (only one answer)

1. $f(x) = \pi + \frac{2}{\pi} \sum_{j=0} (j+1)^{-2} \cos{(j+1)x}$ 2. $f(x) = 1 - \frac{1}{\pi} \sum_{j=0} (4j+1)^{-2} \sin(4j+1)x$
3. $f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{j=0} (j+1)^{-2} \sin(j+1)x$ 4. $f(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{j=0} (2j+1)^{-2} \cos{(2j+1)x}$

1.6 In the double-slit experiment, the slits are spaced by *d* and the wavelength of the light is $\lambda$. We see interference fringes on the screen located far away from the slits. What are the incorrect statements?

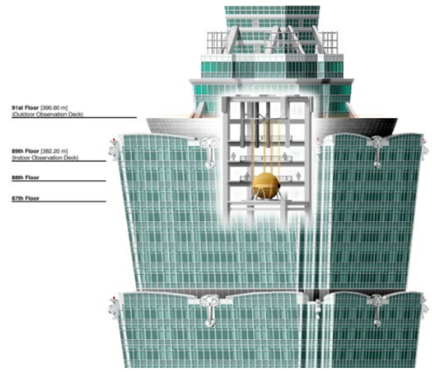
1. As long as the screen is large enough, one can find infinite number of fringes.
2. There are only finite number of fringes.
3. If we increase the spacing *d*, there will be denser fringes on the screen.
4. If we increase the wavelength $\lambda$, there will be more fringes.

1.7 Identify the wrong statements about sound waves in air (1 bar, 25 degrees C)

1. When the sound wave propagates to the right, air molecules can be moving in any direction.
2. Even in the presence of the wind (below sound speed), air molecules do not move faster than the sound speed.
3. Snell's law applies to light and sound waves. Huygens' principle and Fermat's principle only apply to light waves.
4. A sound source is moving at 90% of sound speed, you can experience the sonic boom if you are moving faster than 10% of sound speed in the same direction.

**Problem 2 Tuned Mass Damper (TMD) of modern skyscrapers (6 points each)**

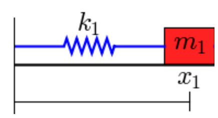
Taipei 101 in Taiwan was the tallest skyscraper in 2004 to 2011. Located in a seismic active zone near the Philippine-Eurasian fault line, the building is protected against earthquake with a tuned mass damper (TMD), see Figure. Watch its action when an earthquake struck: [https://www.youtube.com/watch?v=ohKqE_mwMmo](https://www.youtube.com/watch?v=ohKqE_mwMmo)



We will try to understand how TMD suppresses the impact of the earthquakes. Here are the basic parameters of the system:

*Building height h=509.2 m*
*Building mass* $m_1$*=705, 140 tons*
*Resonance period* $T_1 = 7.5s$
*Estimated quality factor* $Q_1$*=10* $(Q = \omega/\gamma)$
*TMD mass* $m_2$ *=728 tons*

a) We model the building as a point mass $m_1$ located at $x_1(t)$. It is connected to the wall with a spring with force constant $k_1$. Earthquake is modelled as the displacement of the wall by $w(t)$. The motion of the building can be described as

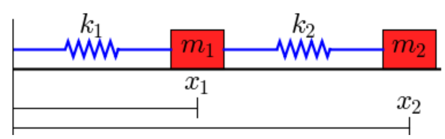



$$x_1''(t) + \gamma_1 x_1'(t) + \omega_1^2 x_1(t) = f(t)$$

Express the damping coefficient $\gamma_1$, spring force constant $k_1$ and external force $f(t)$ in terms of the basic parameters and $w(t)$.

An earthquake strikes the building with unity amplitude $w(t) = e^{i\omega t}$. How large is the shaking amplitude of the building $A_1$? What is the amplitude of the worst-case scenario of resonant driving $\omega = \omega_1$ ? Here and below you can express answers in $\gamma$ and $\omega$.

b) The TMD is modelled as a second point mass $m_2$ that connects to the building with force constant $k_2 \equiv m_2\omega_2^2$ and damping coefficient $\gamma_2$. Write down the equations of motion of both objects. And show that in the presence of the driving $w(t) = e^{i\omega t}$, the shaking amplitudes of the TMD $A_2$ and the amplitude of the building $A_1$ are related by




$$A_2 = \frac{\omega_2^2}{\sqrt{(\omega_2^2 - \omega^2)^2 + \omega^2\gamma_2^2}} A_1$$

(Hint: since $m_2 \ll m_1$, you may keep $m_2/m_1$ to leading order.)

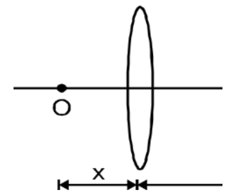
c) Show that it is possible to employ the TMD to fully eliminate the shaking amplitude of the building $A_1 = 0$ subject to the resonant driving $w(t) = e^{i\omega_1 t}$. What are the values of $\omega_2$ and $\gamma_2$ to realize such condition, and what would be the shaking amplitude of the TMD $A_2$?

d) Earthquakes come with all frequencies. One generic strategy is to set $\omega_2 = \omega_1$ and tune the damping coefficient of the TMD to an optimal value $\gamma_2 = \gamma^*$. Show that this scheme can effectively increase the damping near the resonance $\omega = \omega_1$ as

$$\gamma_{eff} \approx \gamma_1 + \frac{m_2}{m_1}\frac{\omega_1^2}{\gamma^*}$$

**Problem 3 Geometrical Optics (6 points each)**

Compound lenses are widely used in optical instruments. Here we will study the general properties of single-lens and two-lens system.

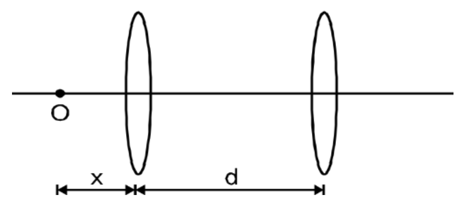



Single lens: Consider a point light source is placed at the origin O and the lens of focal lens is located at x away from the source. Use the Lens equation $\frac{1}{D_0}+\frac{1}{D_i}=\frac{1}{f}$, where $D_0$ is the distance to the object and $D_i$ is the distance to the image and show that

a) For a convex lens with $f>0$, show that we get a real image only when $x>f$ and the magnification is $M=-\frac{D_i}{D_0}=\frac{f}{f-x}$.
Draw light rays for case of $x=\frac{4}{3}f$ and show that you get a real image at $D_i=3f$ with the magnification of $M=-3$.

Hint: real image means the light can form an image on the screen, which requires $D_i>0$.

b) Now we move on to two-lens system, where the second lens with focal length $f'$ is placed $d$ away from the first one, see figure. Taking the image of the first lens as the object for the second lens. Determine the object distance $D_o'$ and the image distance $D_i'$ with respect to the second lens. Show that the overall magnification is




$$MM'=\frac{D_i}{D_0}\frac{D_i'}{D_0'}=\frac{ff'}{(f-x)(f'-d)-xf}.$$

(Comment: A special case, called infinite conjugation, which we touched on in HW7, can now be understood as the situation that the overall magnification does not depend on the distance between the lenses. Such condition leads to $MM'=\frac{f'}{f}$.)

c) In the case of $d \ll x$, the two lenses can be considered as one compound lens with an effective focal length $f_{eff}$. Show that

$$f_{eff}=\frac{ff'}{f+f'}.$$

d) Based on the above, explain in words why when you watch something with a magnifier, the image appears larger and non-inverted when the magnifier moves away from the object and, suddenly the image becomes upside down and starts shrinking when the magnifier gets too far beyond a critical separation $x>x^*$,. Assume lens 1 is the magnifier and lens 2 is (one of) your eyes. Determine the distance $x^*$ that the image suddenly becomes upside down?
(Remark: If you have not seen this effect, try it with the lens from the Physics Depart. Remember that images on our retina is also up-side down. Finally, if you know $x^*$ and the focal length of the magnifier $f$, you can calculate the focal length of your eyes.)

**Problem 4 Thermodynamical process and heat engine (6 points each)**

Consider an ideal monatomic gas initially prepared in a container of volume $V_0$. Its initial pressure $P_0$ and temperature $T_0$. Consider the following cycling processes and express all your results in terms of $V_0$, $P_0$ and $T_0$.

Step 1: the system isothermally compressed by a factor of 2 in volume.
Step 2: the system is cooled isochorically until it recovers its initial pressure $P_0$.
Step 3: the system is heated isobarically until it recovers its initial volume $V_0$.

a. Draw a P-V diagram and schematically draw the cycle that describes the 3 steps.

b. Calculate the internal energy U, pressure P, volume V and temperature T in the beginning of each step. Fill the table below

| Step | Internal energy U | Pressure P | Volume V | Temperature T |
|---|---|---|---|---|
| 1 | $\frac{3}{2}P_0V_0$ | $P_0$ | $V_0$ | $T_0$ |
| 2 | | | | |
| 3 | | | | |

c. List the amount of work the system does to the environment and the heat the system absorbs from the environment during each step. (Hint: use 1st law energy conservation)

| Step | Work done $\Delta W = \int PdV$ | Heat absorbed $\Delta Q$ | Entropy increased $\Delta Q$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |

d. Calculate the total amount of work the system does to the environment, the total amount of heat absorbed by the system and total change of the entropy after one full cycle.

e. Does the system function more like a heat engine or a refrigerator?